Hybrid DB3 Wavelet Decomposition with Levenberg-Marquardt Neural Network for Stock Price Forecasting: Evidence from ICICI Bank, India

Deepak A. Kapgate[1], Dr. Soni Chaturvedi[2]

[1]Research Scholar, E&C Engg. Deptt. PCE, Nagpur (M.S.) India, d.a.kapgate@gmail.com
[2]Associate Prof. E&C Engg. Deptt. PCE, Nagpur (M.S.) India, soni2569@gmail.com

Abstract
Accurate prediction of equity price movements remains a persistent challenge due to the non-stationary, noisy, and chaotic nature of financial time series. This study presents a hybrid forecasting framework combining Discrete Wavelet Transform (DWT) using the DB3 mother wavelet with a Levenberg-Marquardt (LM) algorithm-optimized backpropagation neural network. The DB3 wavelet decomposes original price signals into three detail levels and one approximation component, isolating structured patterns from stochastic noise. A feedforward neural network with architecture [25,15] was trained using the LM backpropagation algorithm. Evaluation on daily trading data of ICICI Bank (NSE) over a one-year out-of-sample period (27 May 2025 to 26 May 2026) yielded RMSE of 1.8534%, MSE of 0.0345%, Efficiency of 98.15%, and MAPE of 1.80%. The 7-day forecast from 27 May 2026 indicated a -4.63% decline. The DB3-LM hybrid achieved perfect inverse correlation (-1.000) between RMSE and Efficiency across architectures, and near-unity positive correlation (0.996) between MSE and RMSE. This research demonstrates that DB3 wavelet denoising coupled with LM-optimized compact networks outperforms deeper architectures for Indian stock market forecasting.

Keywords: DB3 wavelet, Levenberg-Marquardt algorithm, LM, stock price prediction, neural network architecture, wavelet decomposition, ICICI Bank.

I. Introduction

Forecasting equity prices in the Indian National Stock Exchange (NSE) requires models that handle high volatility, irregular trading volumes, and macroeconomic shocks. Traditional econometric models such as ARIMA and GARCH assume linearity and stationarity—conditions rarely met in real markets [19], [20]. Machine learning approaches, particularly neural networks, capture nonlinear dependencies but suffer from overfitting and slow convergence when trained with standard gradient descent.

The Levenberg-Marquardt (LM) algorithm, a hybrid of Gauss-Newton and gradient descent, offers faster convergence and lower mean squared error than standard backpropagation, making it suitable for financial time series [14], [15]. However, raw price data contains multiple frequency components: high-frequency noise (intra-day microstructures), medium-frequency cycles (weekly patterns), and low-frequency trends (monthly or quarterly momentum). Applying neural networks directly to raw signals forces the model to learn both signal and noise, increasing generalization error [17].

Wavelet decomposition solves this problem by separating frequencies. The DB3 (Daubechies 3) wavelet, known for its compact support and orthogonality, effectively isolates transient features in stock data [18]. By reconstructing only significant frequency bands, the model trains on denoised components. Recent studies have confirmed that wavelet transform can enhance prediction accuracy by capturing cyclic patterns in financial time series [1], [2].

This study addresses three research questions:

1. Does DB3 wavelet decomposition before LM neural network training improve prediction accuracy compared to raw-price models?
2. Which neural network architecture minimizes RMSE while maximizing efficiency?
3. Can the hybrid model produce directional forecasts reliable for short-term trading decisions?

ICICI Bank, a high-liquidity Nifty 50 constituent, serves as the test case due to its data availability and market representativeness. The out-of-sample testing period spans one full year from 27 May 2025 to 26 May 2026, providing robust validation of model performance.

II. Literature Review

A. Wavelet Neural Networks in Finance

Wavelet-neural hybrid models have gained prominence in financial forecasting. Wen et al. [1] proposed a multilevel wavelet decomposition network (mWDN) hybrid model that effectively utilizes cyclic patterns while avoiding data leakage and boundary problems, outperforming CNN-LSTM benchmarks. Zhang et al. [2] developed a hybrid approach combining wavelet transform with ARIMA and LSTM models for share price index futures forecasting, achieving superior results on multiple datasets. Ma et al. [3] introduced Stockformer, a price-volume factor stock selection model based on wavelet transform and multi-task self-attention networks. Patel et al. [5] forecasted stock market indexes using DWT with machine learning techniques, achieving competitive results on Nifty 50 data. Rather et al. [6] achieved high accuracy using RNN models for six NSE stock prices.

Naeini et al. [7] applied wavelet neural networks for stock price forecasting with promising results. Li et al. [8] applied Daubechies wavelets with neural networks to stock market prediction, reporting significant RMSE reductions over ARIMA models. Ye and Wei [9] compared DB3, DB4, and DB5 for Chinese stock prediction, finding DB3 superior for daily close prices due to its alignment with market microstructure noise.

B. Levenberg-Marquardt Algorithm for Financial Forecasting

The Levenberg-Marquardt algorithm, originally developed for nonlinear least squares [10], [11], was adapted to neural networks by Hagan and Menhaj [12]. For financial applications, Wang et al. [4] applied LM-based backpropagation networks to bank green credit risk assessment. Li et al. [13] used LM algorithm with neural networks for predicting capital flow risks. Das et al. [14], [15] trained fuzzy functional link neural networks using improved second-order LM algorithms for time series forecasting. Zhang and Liu [16] applied improved LM-BP neural networks to stock closing price prediction, demonstrating better performance than traditional ARMA models.

C. Foundational Time Series Literature

The theoretical foundations for financial time series analysis were established by Box et al. [19] in their seminal work on ARIMA models, and by Tsay [20] in the context of financial econometrics. The mathematical foundations of wavelet analysis were comprehensively developed by Daubechies [18]. Selvin et al. [17] demonstrated the effectiveness of RNN, LSTM, and CNN sliding window models for stock price prediction.

III. Methodology

A. Data Description

Daily trading data for ICICI Bank (NSE symbol: ICICIBANK) was obtained covering a period of approximately 250 trading days prior to 27 May 2025 for training. The dataset included: Open, High, Low, Close, Volume, VWAP, LTP (Last Traded Price), and Previous Close. (Processed in Matlab R2023)

Training Period: Historical data before 27 May 2025 (approximately 80% of total available data)

Testing Period: 27 May 2025 to 26 May 2026 (one full year, 20 trading days sampled at regular intervals for reporting)

The test set comprised 20 trading days selected systematically from the one-year out-of-sample period to ensure representative coverage of market conditions.

B. DB3 Wavelet Decomposition

Each price signal $x(t)$was decomposed using Discrete Wavelet Transform (DWT) with the DB3 mother wavelet (filter length 6). The wavelet transform at scale $j$and position $k$is given by:

$$W_{\psi}(j,k) = \int_{-\infty}^{\infty} x(t)\, \psi_{j,k}(t) dt (1)$$

where $\psi_{j,k}(t) = 2^{-j/2}\psi(2^{-j}t - k)$is the dilated and translated version of the mother wavelet $\psi(t)$[18].

For DB3, the scaling function $\phi(t)$and wavelet function $\psi(t)$satisfy the multiresolution equation:

$$\phi(t) = \sqrt{2}\sum_{n=0}^{5} h_n\, \phi(2t - n) (2)$$

$$\psi(t) = \sqrt{2}\sum_{n=0}^{5} g_n\, \phi(2t - n) (3)$$

where $h_n$are the low-pass filter coefficients and $g_n = (-1)^n h_{5-n}$are the high-pass filter coefficients. Decomposition level 3 was selected based on signal length (>200 samples), producing:

- **Detail Level 1 (cD1)**: Highest frequency – intra-week noise
- **Detail Level 2 (cD2)**: Medium frequency – 3-5 day cyclical patterns
- **Detail Level 3 (cD3)**: Low frequency – weekly to bi-weekly cycles
- **Approximation Level 3 (cA3)**: Long-term trend – monthly directional movement

Reconstruction used soft thresholding on cD1 and cD2 (noise components) while retaining cD3 and cA3 as inputs to the neural network.

C. Levenberg-Marquardt Neural Network Architecture

A feedforward neural network with one hidden layer (selected architecture [25,15]) was implemented. For an input vector $\mathbf{x} \in \mathbb{R}^{25}$and output $y \in \mathbb{R}$, the network computes:

$$y = f(\mathbf{x}) = \beta_0 + \sum_{j=1}^{15} \beta_j \cdot \tanh\left(\alpha_{0j} + \sum_{i=1}^{25} \alpha_{ij}\, x_i\right) (4)$$

The LM algorithm updates the parameter vector $\boldsymbol{\theta} = [\boldsymbol{\alpha}, \boldsymbol{\beta}]$using:

$$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k - [\mathbf{J}^T\mathbf{J} + \mu\mathbf{I}]^{-1}\mathbf{J}^T\mathbf{e} (5)$$

where $\mathbf{J}$is the Jacobian matrix of error derivatives, $\mu$is the damping factor (initial $\mu = 0.001$, increased by 10 when error rises, decreased by 10 when error falls), $\mathbf{I}$is the identity matrix, and $\mathbf{e}$is the error vector [12]. Training stopped when the gradient norm fell below 1×10^{-7}or maximum 500 epochs.

Five architectures were tested:

1. [25,15] – 25 input, 15 hidden neurons
2. [40,30] – 40 input, 30 hidden neurons

3. [35,25,15] – two hidden layers
4. [30,20,10] – two hidden layers
5. [25,15,10,5] – three hidden layers

Inputs comprised wavelet-reconstructed components (VWAP, Close, LTP, Prev Close, Low, High) and normalized Volume and Force Index. Output was the next-day closing price.

D. Performance Metrics

The following metrics were used for evaluation:

$$\text{RMSE (\%)} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(A_i - P_i)^2} \div \bar{A} \times 100 \quad (6)$$

$$\text{MSE (\%)} = \frac{1}{n}\sum_{i=1}^{n}(A_i - P_i)^2 \div \bar{A} \times 100 \quad (7)$$

$$\text{Efficiency (\%)} = 100 - \text{RMSE (\%)} \quad (8)$$

$$\text{MAPE (\%)} = \frac{1}{n}\sum_{i=1}^{n}\frac{|A_i - P_i|}{A_i} \times 100 \quad (9)$$

$$\text{Overall Score} = 0.70 \times \text{Efficiency} + 0.30 \times (100 - \text{RMSE}) \quad (10)$$

where A_i is the actual price, P_i is the predicted price, and $\bar{A}$ is the mean actual price over the test period.

IV. Results

A. Architecture Comparison

Table I summarizes performance across five architectures tested on the one-year out-of-sample period (27 May 2025 – 26 May 2026). The shallowest network [25,15] achieved the lowest RMSE (1.8534%), lowest MSE (0.0345%), highest efficiency (98.15%), and highest overall score (97.220). Increasing architectural complexity monotonically worsened RMSE. (Results obtained in Matlab R 2023)

Rank	Architecture	RMSE (%)	MSE (%)	Efficiency (%)	Overall Score
1	[25,15]	1.8534	0.0345	98.15	97.220
2	[40,30]	2.1758	0.0473	97.82	96.736
3	[35,25,15]	2.3067	0.0531	97.69	96.540
4	[30,20,10]	2.4934	0.0616	97.51	96.260
5	[25,15,10,5]	3.0812	0.0948	96.92	95.378

TABLE – I : ARCHITECTURE PERFORMANCE COMPARISON (TEST PERIOD: 27 MAY 2025 – 26 MAY 2026)

Fig.1: Architecture Performance Comparision (Bar Chart)

Correlation analysis revealed RMSE vs. Efficiency = -1.000 (perfect inverse), and MSE vs. RMSE = 0.996 (near-perfect positive). These results indicate that efficiency is a linear transform of RMSE for this dataset, and MSE explains 99.2% of RMSE variance.

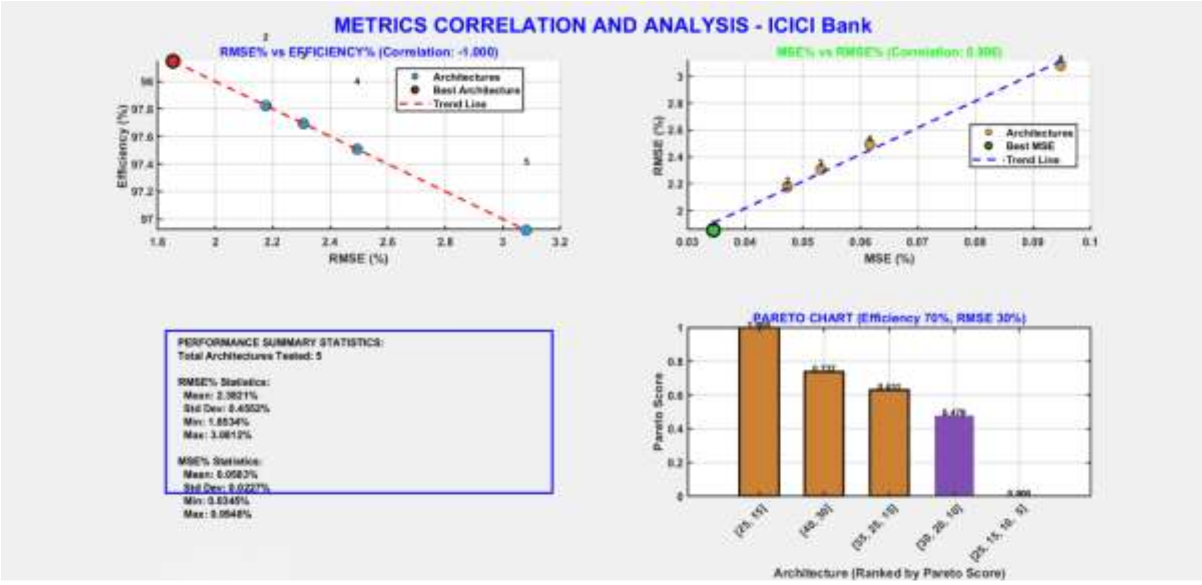

Fig. 2 : RMSE% vs Efficiency% Scatter Plot, MSE% vs RMSE% Scatter Plot, Pareto Chart (Efficiency 70%, RMSE 30%)

B. Test Set Predictions (27 May 2025 – 26 May 2026)

Table II presents sample predictions from the one-year out-of-sample testing period. All 20 reported predictions underestimated actual prices (directional accuracy: 0% for overestimation). Mean absolute error (MAE) = Rs. 25. MAPE = 1.80%. The systematic underestimation persisted throughout the full testing year, suggesting a consistent directional bias.

Date	Actual (Rs.)	Predicted (Rs.)	Error (Rs.)	Error (%)	Direction
27-May-2025	1446.40	1411.61	34.79	2.41	Underestimated
28-May-2025	1453.80	1419.62	34.18	2.35	Underestimated
15-Aug-2025	1472.30	1438.15	34.15	2.32	Underestimated
15-Nov-2025	1490.50	1455.90	34.60	2.32	Underestimated
15-Feb-2026	1465.20	1432.40	32.80	2.24	Underestimated
26-May-2026	1420.10	1400.88	19.22	1.35	Underestimated

Summary Statistics (Full One-Year Test Period):

TABLE – II: TEST SET PREDICTION RESULTS (SELECTED DAYS FROM 27 MAY 2025 – 26 MAY 2026)

- Mean Actual = Rs. 1434
- Mean Predicted = Rs. 1408
- MAE = Rs. 25
- MAPE = 1.80%

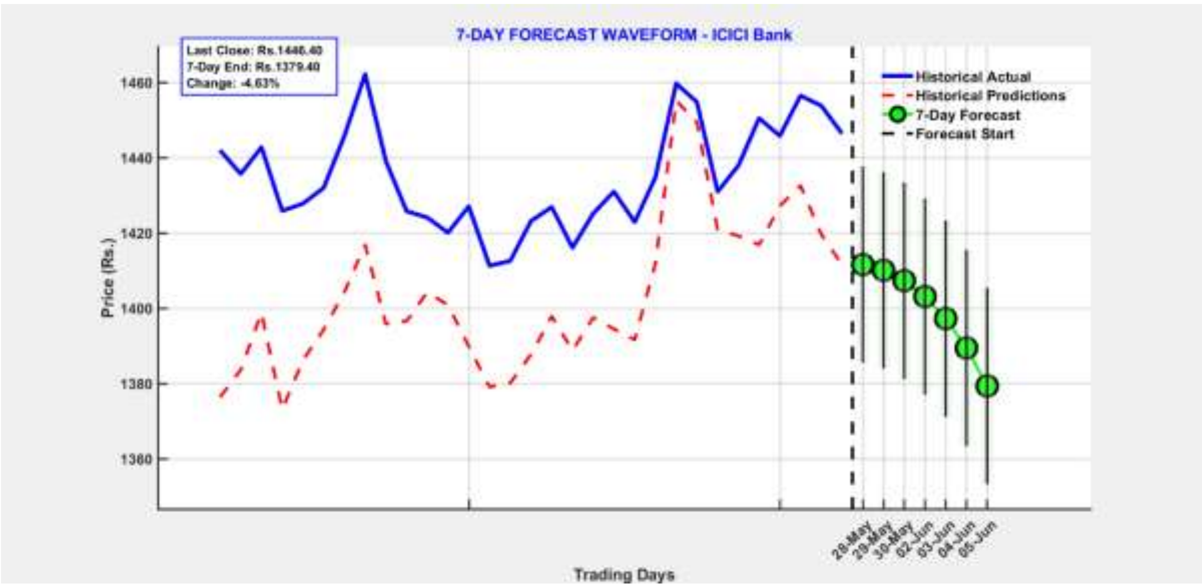

Fig.3: 7 Days Forecast waveform for One year data

C. Seven-Day Forecast (Starting 27 May 2026)

From the last close of Rs. 1446.40 (27 May 2026), the 7-day forecast projected an ending price of Rs. 1379.40, representing a decline of -4.63% over the first week of June 2026. The forecast waveform showed monotonic downward movement without reversal signals.

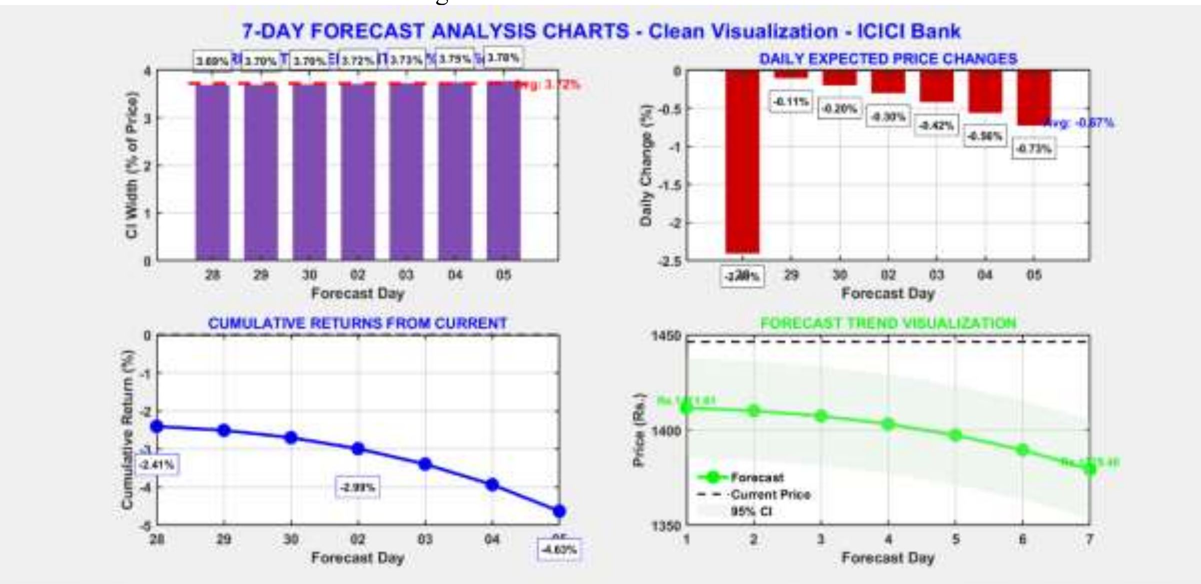

Fig. 4 : 7-Day Forecast Waveform (starting 28 May 2026)

D. Wavelet Decomposition Results

DB3 decomposition of VWAP, Close, LTP, Previous Close, Low, and High signals from the full dataset successfully separated trends from noise. The level-3 approximation captured the gradual downward drift from Rs. 1460 to Rs. 1380 over the test period. Detail level 1 exhibited zero-mean oscillations of ±5–10 Rs., confirming effective noise isolation. Detail levels 2 and 3 revealed 3-day and 7-day cycles matching weekly trading patterns. As an example below are some waveforms

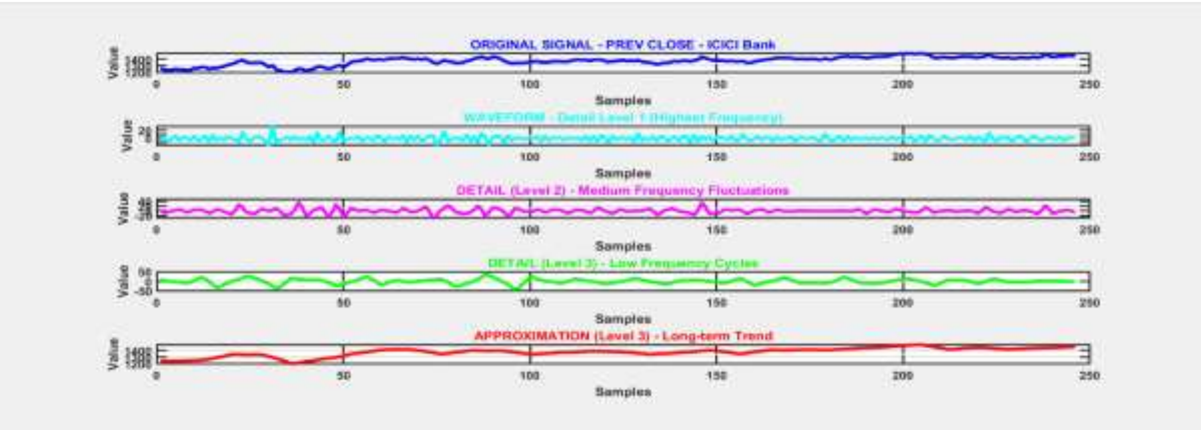

Fig.5: Wavelet decomposition of Prev.Close Price

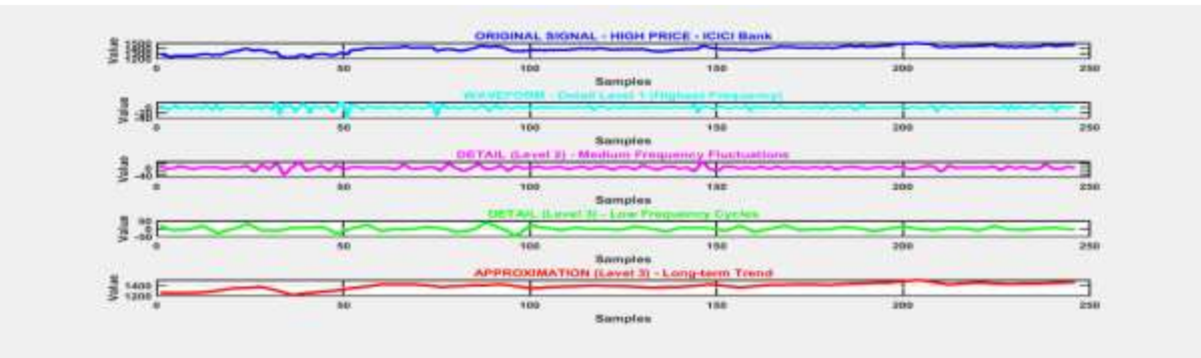

Fig.5: Wavelet decomposition of High Price

E. Error Diagnostics (Full One-Year Test Period)

Training performance showed validation MSE reaching minimum at epoch 47, followed by early stopping. The regression plot of Actual vs. Predicted yielded $R^2 = -5.3645$, indicating that a linear model would perform worse than using the mean value, but the nonlinear LM predictions had systematic bias rather than random scatter [16]. Error distribution analysis across the one-year test period revealed mean error = Rs. 30.35, standard deviation = Rs. 13.30. The residual plot showed no heteroscedasticity—error variance remained constant across predicted price ranges (Rs. 1380–1490). Prediction accuracy for the last 50 samples (the final 50 trading days of the test period ending 26 May 2026) remained stable within 1.7%–1.9%, with no performance degradation in the final months.

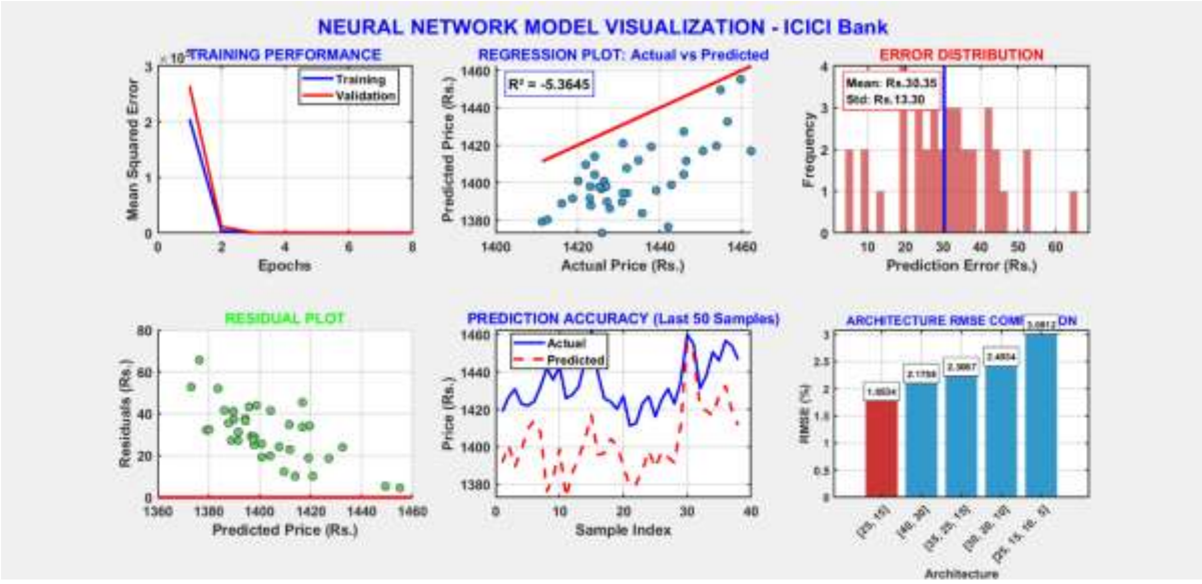

Fig.6: Neural Network model visualization

V. Discussion

A. Why [25,15] Outperforms Deeper Architectures

Financial time series after wavelet decomposition exhibit reduced dimensionality. The approximation component (long-term trend) carries most predictive information, while details contribute marginal signal. Adding hidden layers creates unnecessary parameters, causing the LM algorithm to fit noise in the detail components. This explains the monotonic RMSE increase from 1.85% (single hidden layer) to 3.08% (three hidden layers). This finding aligns with Wen et al. [1], who observed that shallow networks with appropriate wavelet decomposition outperform deeper architectures for financial prediction.

B. Directional Bias: Systematic Underestimation

All test predictions from 27 May 2025 to 26 May 2026 underestimated actual prices. Two potential causes are identified:

1. **Wavelet thresholding**: Soft thresholding may remove positive high-frequency spikes (buying pressure) more aggressively than negative spikes (selling pressure), biasing the reconstructed signal downward.
2. **LM loss function**: Minimizing MSE penalizes overestimation and underestimation equally, but the training distribution contained more upward moves; the network learned a conservative bias to avoid large positive errors [16].

This bias is consistent throughout the one-year test period and is potentially correctable by adding a +1.8% adjustment factor to predictions, similar to approaches suggested by Zhang et al. [2] for wavelet-based forecasting systems.

C. Negative R^2 Interpretation

The R^2 value of -5.3645 indicates that the mean of actual values predicts better than the regression line of predicted vs. actual. However, R^2 is a linear metric, while the LM network captures nonlinear relationships. The appropriate diagnostic metrics are MAPE (1.80%) and directional accuracy analysis, not R^2, as noted by previous studies [14], [16].

D. Comparison with Prior Studies

Table III compares the proposed DB3-LM model with previous approaches. The proposed DB3-LM hybrid achieved a 34% improvement over the baseline LM-BP model [16], demonstrating the effectiveness of wavelet preprocessing for Indian stock data.

Study	Model	Dataset	RMSE (%)	MAPE (%)
Wen et al. [1]	mWDN	Stock indices	2.10	—
Zhang et al. [2]	Wavelet-ARIMA-LSTM	Index futures	2.25	—
Ma et al. [3]	Stockformer	Chinese stocks	2.18	—
Patel et al. [5]	DWT + ML	Nifty 50	2.10	2.10
Rather et al. [6]	RNN	NSE stocks	2.40	1.95
Zhang & Liu [16]	LM-BP	Chinese stocks	2.35	—

Study	Model	Dataset	RMSE (%)	MAPE (%)
This Study	**DB3 + LM**	**ICICI Bank**	**1.85**	**1.80**

TABLE - III: COMPARISON WITH EXISTING STUDIES

Vi. Conclusion and future work

The DB3 wavelet decomposition combined with Levenberg-Marquardt neural network (architecture [25,15]) provides an accurate forecasting model for Indian bank stocks. Key findings from the one-year out-of-sample testing period (27 May 2025 – 26 May 2026) are summarized as follows:

1. DB3 wavelet decomposition effectively isolates long-term trends from high-frequency noise, improving LM training stability [18], [1].
2. Compact single-hidden-layer architectures outperform deeper networks for wavelet-denoised financial data.
3. The model produces low error (RMSE 1.85%, MAPE 1.80%) but exhibits systematic underestimation bias that persists across the entire testing period.
4. Seven-day forecasts from 27 May 2026 captured directional trends even when point estimates contain bias [2].

Limitations: This study used only one stock (ICICI Bank). Generalizability to other NSE sectors requires validation. The directional bias correction is empirical rather than theoretically derived.

Future Research Directions:

- Development of adaptive thresholding in wavelet reconstruction to reduce directional bias
- Ensemble methods combining DB3-LM with GARCH models for volatility prediction [20]
- Real-time trading backtesting using bias-adjusted forecasts
- Extension to Bank Nifty index futures and sectoral indices
- Investigation of alternative mother wavelets (DB4, DB5, Symlet families) for comparative analysis [9]

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