Sustainable Inventory Models with Trade Credit Option in Fuzzy Environment

Subhashree Parida[1], Milu Acharya[2*]

Abstract

Currently, global warming is the most challenging issue for all countries, and it is a byproduct of greenhouse gas emissions. Because of this, researchers across various fields are developing green practices to control emissions, ultimately reducing harmful impacts on the climate. Again, the present study includes a cost for waste disposal purposes. Again, the research suggests sustainable two-warehouse deteriorating models under partial back-logging for both crisp and intuitionistic fuzzy conditions, where the models are formed for a time-dependent demand function with a single trade-credit option for the retailer. This study involves optimal investments in green technology to mitigate the GHG effect from the transportation of goods. The optimal total cost, along with algorithms for the solution process, is proposed for crisp and intuitionistic fuzzy models. Appropriate numerical examples are provided to verify optimal conditions for total cost, and the results are validated through suitable analysis that helps minimize the average total cost under different pricing and ordering policies.

[1]Department of Mathematics, Siksha 'O'Anusandhan (Deemed to be University) Khandagiri, Bhubaneswar – 751030, Odisha, India, Email: subhashreeparida27@gmail.com, Orchid ID: 0009-0002-1181-3130

[2*]Department of Mathematics, Siksha 'O' Anusandhan (Deemed to be University), Bhubaneswar – 751030, Odisha, India, Email: miluacharya@soa.ac.in, Orchid id: 0000-0003-4273-2886

***Corresponding Author:** Milu Acharya, Department of Mathematics, Siksha 'O' Anusandhan (Deemed to be University), Bhubaneswar – 751030, Odisha, India, Email: miluacharya@soa.ac.in, Orchid id: 0000-0003-4273-2886

Keywords: Carbon emissions, Green investment, Cost for disposal, Trade credit, Intuitionistic fuzzy, α,β - cut method.

1. Introduction

CO2 is the main driver of environmental harm from industry. This emission acts as a blanket over the environment, trapping heat and warming the Earth's surface, which affects crop production, weather conditions, and sea levels. Regulatory bodies impose appropriate carbon taxes on the industrial sectors to control the weather conditions. Given this, industrial sectors invest in green technology (GTI), an outcome of scientific innovation aimed at mitigating the harmful effects of emissions. Inventory control and supply chain management researchers implement carbon taxes and GTI to formulate economic order quantity (EOQ) and economic production quantity (EPQ) problems. Lin et al. [1] proposed "a model for perishable items with a two-stage pricing policy in the selling process. "An optimum sustainable inventory model with backorder and deterioration under controllable carbon emissions has been proposed by Mishra, Wu, and Sarkar [2]. "A sustainable deteriorating model with controllable carbon emission using green technology (GT) was suggested by Pervin [3]." "A sustainable model for a two-echelon cold chain with GTI and stock-dependent demand under carbon emissions tax regulation, presented in Sebatjane [4]." Muthusamy, Murugesan, and Selvaraj [5] considered "an optimal production model with shortage for deterioration item and effect of carbon emission policy combination with GT."

Again, to avoid space constraints, retailers prefer multiple warehouses for storing goods. Mostly, they are considered to be the own warehouse (OWH) and the rented warehouse (RWH), with different storage charges. "An optimization model in a green environment with two warehouses has been discussed by Motla et al. [6]." Kumar, Dahiya, and Kumar [7] studied "a two-warehouse model for deteriorating items with fixed shelf-life stock-dependent demand and partial backlogging."

Classical dynamical systems deal with fixed, deterministic demand rates and known parameter values. In reality, uncertainties in demand and costs are present in the system. Hence, when formulating EOQ and EPQ models, several researchers preferred fuzzy numbers to account for uncertainties arising from market conditions. In the present era, new ideas in business environments are highly required to strengthen the backbone of the sustainability of goods. In business propagation, one such adoption is the trade credit facility between the supplier and their retailers. Under trade credit terms, the supplier offers the retailer the option to delay payment, creating a time gap between when the units are received and when payment is made. During that gap, the supplier levies no interest on the retailer. This facility helps both parties: the retailer gains higher profits from a higher volume of orders and benefits from lower unit purchasing costs due to bulk purchases. Again, the retailer also earns interest on the revenue generated from the late payment. But as soon as the payment falls outside the specified late payment period, the retailer pays the interest set by the supplier. Chen [8] discussed "an integrated assembled production decision support inventory system with permissible delay in payment option." Cardenas Baroon et al. [9] proposed "an EOQ model with nonlinear stock-dependent holding cost, nonlinear stock-dependent demand, and trade credit." Momena et al. [10] mentioned "a two-storage model with a trade credit policy and time-varying holding cost under quantity discounts."

Dubey and Kumar [11] formulated "the classical model under an uncertain environment." Intuitionistic fuzzy (IF) sets are used in models to address uncertainties in demand and costs. "A study of an EOQ model of green items with the effect of carbon emission under pentagonal intuitionistic dense fuzzy environment, as mentioned in Maity et al. [12]. "Intuitionistic fuzzy model with quadratic demand rate, time-dependent holding cost and shortages discussed in Chaudhary and Kumar [13] ." Singh and Kumar [14] discussed "an IF model with waste disposal using TIFN."

1.1 Research Gap

Considering the research related to two-warehouse models, mentioned in Table 1, we observe the following research gaps

i) No investment for disposing of the deteriorated items.
ii) No trade-credit facility is considered.
iii) No uncertainty is expected related to different costs.

1.2 Contribution to the present research

(i) Two-warehouse model with linear demand depending on time and constant deterioration rates.
(ii) Costs involved in the problem to minimize the total cost: cost for disposal, cost due to emission, cost towards holding for OWH and RWH, transportation cost, order cost, and interest paid/earned because of trade credit facilities.
(iii) Formulation of a model with IF parameters used for unit-wise disposal, holding, and purchase costs.

Table 1 shows the difference between earlier research, including this work.

Table 1. Selected author's contribution

Author(s)	Two-warehouse problem	Time-dependent demand	Deterioration	Disposal cost	Optimum GT investment	The concept of carbon emissions	IF consideration for different costs	Inclusion of trade credit

Mashud et al. [16]	Y	-	Y	-	Y	Y	-	-
Sahoo, Acharya, and Patnaik [17]	Y	Y	Y	-	Y	Y	-	-
Momena et al. [10]	Y	Y	Y	-	-	-	-	Y
Pal and Chakraborty [18]	-	-	-	-	-	-	Y	Y
San-Jose et al. [19]	-	-	Y	Y	-	Y	-	-
Present study	Y	Y	Y	Y	Y	Y	Y	Y

2. Fundamentals of Fuzzy Sets

Fuzzy set: Zadeh [15] stated that "fuzzy set A of a universal set X to be a set of ordered pairs $\{(x, \mu_A(x)) : x \in X\}$, where $\mu_A(x) : x \mapsto y \in [0,1]$, which assigns a real number y in $[0,1]$ to each element $x \in X$, where the function $\mu_A(x)$ is the membership function of the fuzzy set A, which is a continuous mapping from X to the closed interval $[0,1]$, and A is a convex set having its α-cut of A_α, which is a set of real numbers and it is expressed as: $A_\alpha = \{x : \mu_{\tilde{A}}(x) \geq \alpha : 0 \leq \alpha \leq 1\}$."

Intuitionistic Fuzzy Number(IFN) (Singh and Kumar [14]): "Let X be a non-empty set of discourses. B be an intuitionistic fuzzy set in X given by ordered triples of an object x, where the degree of belongingness is $\mu_B(x)$ and the degree of non-belongingness is $v_B(x)$ of B. Intuitionistic fuzzy number is defined by $B = \left\{\left\langle x, \mu_B(x), v_B(x) \right\rangle : x \in X\right\}$, where $\mu_B(x), v_B(x) : X \to [0,1]$ for every $x \in X_B$ with $0 \leq \mu_B(x) + v_B(x) \leq 1$. The value of $\pi_B(x) = 1 - \mu_B(x) - v_B(x)$ is called the IF index of $x \in U$ for B."

Triangular Intuitionistic Fuzzy Number (TIFN) (Singh and Kumar [14]): "TIFNs are a special type of IFN. Let Bt be a triangular Intuitionistic fuzzy number defined by $Bt = \left(b_1, b_2, b_3; b_1', b_2, b_3'\right)$, where $b_1' \leq b_1 \leq b_2 \leq b_3 \leq b_3'$ on $\mathbb{R}$. The following equation, given in (1), represent the membership function $\mu_{Bt}(x)$ and the non-membership function $v_{Bt}(x)$ respectively of the If number Bt."

$$\mu_{Bt}(x) = \begin{cases} \dfrac{x - b_1}{b_2 - b_1}, b_1 \leq x \leq b_2 \\ \dfrac{b_3 - x}{b_3 - b_2}, b_2 \leq x \leq b_3 \\ 0, \ otherwise \end{cases}, \quad v_{Bt}(x) = \begin{cases} \dfrac{b_2 - x}{b_2 - b_1'}, \ b_1' \leq x \leq b_2 \\ \dfrac{x - b_2}{b_3' - b_2}, \ b_2 \leq x \leq b_3' \\ 1, \ otherwise \end{cases} \tag{1}$$

Defuzzification of TIFN by $\alpha\beta$ -cut(Singh and Kumar [14]): "Let $Bt = (b_1, b_2, b_3; b_1', b_2, b_3')$ where $b_1' \leq b_1 \leq b_2 \leq b_3 \leq b_3'$ on $\mathbb{R}$. The α −cuts of the membership function $\mu_{Bt}(x)$, for $\alpha \in [0,1]$.

$B(\alpha) = [b_1 + \alpha(b_2 - b_1), b_3 - \alpha(b_3 - b_2)] = [B_L(\alpha), B_R(\alpha)]$ and $\beta -$ cut of non-membership function $v_{Bt}(x)$ for $\beta \in [0,1]$, we get $B(\beta) = \left[b_2 - \beta(b_2 - b_1'), b_2 + \beta(b_3' - b_2)\right] = [B_L(\beta), B_R(\beta)]$.

The following equation is used for defuzzification

$$D_f(B_t) = \frac{1}{2}\left[D_f\left(\mu_{B_t}(x)\right) + D_f\left(\nu_{B_t}(x)\right)\right] = \frac{1}{2}\left[b_2 + \frac{1}{4}\left(b_1 + b_1' + b_3 + b_3'\right)\right] \quad (2)$$

represents the defuzzification formula for TIFNs."

Table 2. Notations used in the models

Notation	Used for	Notation	Used for
O_{rq}	Number of units ordered	a_2	Shape parameter
T_{rc}	Transportation costs in reduced form	c_p	Cost towards purchasing
t_m	Least cost towards transportation	$H_r / \tilde{H}_r$	Cost towards Holding in RWH (for crisp/IF)
d_t	Distance covered from OWH to the customer via RWH	$H_o / \tilde{H}_o$	Cost towards Holding in OWH (for crisp/IF)
F_p	Fuel charge	C_w	Storage capacity of OWH
C_{fc}	Consumption of fuel for the unfilled truck	α'	The deterioration rate meant for RWH
S_{fc}	Extra fuel consumed for each ton of stock	λ'	The deterioration rate meant for OWH
w_p	Weight of each unit item	μ'	Deterioration-free period
T_N	Total trips completed	C_d	Cost meant for disposal (crisp)
c_{e1}	Emission cost due to the movement of a truck	$\tilde{C}_d$	Cost meant for disposal (IF)
c_{e2}	Extra emission cost for transporting one unit of an item	M_T	Credit time for the retailer
a_1	Known Market size	I_{ER}	Interest earned through the retailer
P_{oc}	Cost towards Ordering	I_{PR}	The interest paid by the retailer
Decision Variables (DVs)			
g_c	Investment for GT	T_{sp}	One complete cycle time
A_{tc}	Annual total cost(crisp)	$\tilde{A}_{tc}$	Annual total cost(IF)

3. Model Formulation

Assumptions used for the models:

i) Shortages are not allowed.

ii) A two-green-warehouse model is proposed in which we assume that there is an optimum investment towards green technology in the optimum cycle of time, and finally, under the other assumptions, as given below, we derive the minimum total cost under the uncertain environmental condition.

iii) Retailer gets a credit time M_T from the supplier, where either $0 \le M_T \le T_{sp}$ or $M_T \ge T_{sp}$. Hence, the following are the possible interest rates (earned (I_{ER})/paid (I_{PR})) for the retailer.

- When $0 \le M_T \le T_{sp}$, the interest earned at a rate of I_{ER}.
- When $M_T \ge T_{sp}$, interest paid at a rate of I_{PR}.

iv) The non-deterioration interval is $[0, \mu']$, and deterioration rate α' for OWH and λ' in RWH, both are taken to be constant in the interval $[\mu', T_{sp}]$ as mentioned in Tiwari, Ahmed, and Sarkar [20].

v) Demand level is $D = (a_1 + a_2 t)$, a_1 is the scale parameter and a_2 shape parameter.

vi) "Emission drop formula $F = \xi(1 - e^{-\chi g_c})$, retailer's investment (g_c) with an upper limit on GT expenditure. This cost function $F(g_c)$ is continuously differentiable and satisfying $F'(g_c) > 0, F''(g_c) < 0$" as taken by Mashud et al. [16].

vii) H_r is higher than H_o as mentioned in Shaikh et al. [21].

The level of inventory $I_{RW}(t)$ and $I_{OW}(t)$ in the RWH and OWH are derived by solving equations (3) and (6)-(7) using the conditions mentioned in (4) and (8):

For RWH: $\frac{dI_{RW}}{dt} + \alpha' I_{RW} = -(a_1 + a_2 t), 0 \le t \le \mu'$ (3)

Where, $I_{RW}(0) = C - C_w$ and $I_{RW}(\mu') = 0$ (4)

This implies: $I_{RW}(t) = -\frac{a_1}{\alpha'} + \frac{a_2}{\alpha'^2} - \frac{a_2 t}{\alpha'} + (C - C_w + \frac{a_1}{\alpha'} - \frac{a_2}{\alpha'^2})(1 - \alpha' t),\ 0 \le t \le \mu'$ (5)

For OWH: $\frac{dI_{OW}}{dt} + \lambda' I_{OW} = 0, 0 \le t \le \mu'$ (6)

$\frac{dI_{OW}}{dt} + \lambda' I_{OW} = -(a_1 + a_2 \mu'), \mu' \le t \le T_{sp}$ (7)

Where, $I_{OW}(0) = C_w, I_{OW}(T_{sp}) = 0$ (8)

The solution is: $I_{OW}(t) = C_w e^{-\lambda' t},\ 0 \le t \le \mu'$ (9)

$I_{OW}(t) = \frac{a_1 + a_2 \mu'}{\lambda'} \left[\frac{1 - \lambda' t}{1 - \lambda' T_{sp}} - 1 \right],\ \mu' \le t \le T_{sp}$ (10)

Again, with $I_{RW}(0) = C - C_w$, equation (5) becomes

$C = C_w + \frac{a_1 \mu'}{1 - \alpha' \mu'}$

(11) Through the continuity property at $t = \mu'$ and $t = T_{sp}$, we get

$C_w e^{-\lambda' \mu'} = \frac{a_1 + a_2 \mu'}{\lambda'} \left[\frac{1 - \lambda \mu'}{1 - \lambda' T_{sp}} - 1 \right]$ (12)

Now, per cycle order quantity is $O_{rq} = C$ (13)

Hence, the formulas for the total cost are (Mashud et al. [16]):

1. Cost towards ordering: P_{oc}
2. Cost towards purchasing: $TP_c = c_p C$ (14)
3. Cost towards holding/carrying:

$H_c = H_r[C\mu - C_w \mu' - \frac{\alpha'}{2} C \mu'^2 + \frac{\alpha'}{2} C_w \mu'^2 - \frac{a_1}{2} \mu'^2] + H_o C_w \mu' + H_o (\frac{a_1 + a_2 \mu'}{1 - \lambda' T_{sp}})[\frac{1}{2} T_{sp}^2 + \frac{\mu'^2}{2} - \mu' T_{sp}]$ (15)

4. Cost for Transportation:

It comprises fixed transportation costs, flexible transportation costs, and the CE cost. The CE is estimated by the weight of the item and the number of units ordered, as obtained in Eq. (18), where the size O_{rq} transported from OWH to RWH and then from RWH to customers, for which the retailer pays the government. When the cost towards the CE is calculated, the total distance ($2d_t$) covered by the vehicle is also included. The cost due to emissions is governed by the items delivered O_{rq} known as the truck's capacity. On the whole, T_{nc} is

$\frac{T_N}{T_{sp}}[t_m + (2d_t F_p C_{fc} + d_t F_p S_{fc} w_p O_{rq}) + (2d_t c_{e1} + d_t c_{e2} O_{rq})]$ (16)

5. GT Investment Cost: A preferred GTI cost is calculated by multiplying T_{sp} and g_c: $g_{TC} = g_c T_{sp}$ (17)
6. Cost for transportation in reduced form: The reduction of TC along with the GT investment is: $T_{rc} = \frac{T_N}{T_{sp}}[t_m + (2d_t F_p C_{fc} + d_t F_p S_{fc} w_p O_{rq}) + (2d_t c_{e1} + d_t c_{e2} O_{rq})(1 - \xi(1 - e^{-\chi g_c}))]$ (18)
7. Disposal Cost:

The deteriorated items are eliminated from the lot and sent to a disposal center with a payment towards the disposal of the items, which is given as

$$T_{DC} = C_d[C\alpha'\mu' - C_w \alpha' \mu' - \frac{\alpha'^2}{2} C \mu'^2 + \frac{\alpha'^2}{2} C_w \mu'^2 - \frac{a_1 \alpha' \mu'^2}{2} + C_w \lambda' \mu' + \frac{\lambda'(a_1 + a_2 \mu')}{2(1 - \lambda' T_{sp})}(T_{sp} - \mu')^2] \quad (19)$$

Interest earned (IE) and paid (IP) for the following cases by the retailer (Pervin, Mahata, and Roy [22]): In addition, the supplier offers the retailer a late payment option, which leads to the following two cases.

Case 1: When $0 < \mu' \le M_T < T_{sp}$,

$$IP = I_{PR}(\frac{a_1+a_2\mu'}{\lambda'})\{(\frac{1}{1-\lambda' T_{sp}})(T_{sp}-M_T-\frac{\lambda' T_{sp}^{\ 2}}{2}+\frac{\lambda' M_T^{\ 2}}{2})-T_{sp}+M_T\}$$

$$IE = S_p I_{ER}[\frac{a_1 M_T^{\ 2}}{2}+\frac{a_2 M_T^{\ 3}}{6}] \tag{20}$$

Case 2: When $0<\mu'\le T_{sp}<M_T$,

$$IP = 0$$

$$IE = S_p I_{ER}(M_T-T_{sp})[\frac{a_1 T_{sp}^{\ 2}}{2}+\frac{a_2 T_{sp}^{\ 3}}{6}]]] \tag{21}$$

8. Total cost:

We use Eqs (14), (15), (17) to (20) and the ordering cost to find the total average cost for case 1. Again, Eqs (14), (15), (17) to (19), (21), and the ordering cost are used to find the total average cost for case 2.

$$A_{tc} = \frac{1}{T_{sp}}[P_{oc}+TP_c+H_c+T_{DC}+T_{rc}+g_{TC}+IP-IE] \tag{22}$$

4. Intuitionistic Fuzzy Model

Taking H_{RW}, H_{OW}, d_c and P_c to be TIFN, we propose the following. Here, $\tilde{H}_O=(H_{o_1},H_{o_2},H_{o_3};H'_{o_1},H_{o_2},H'_{o_3})$, $\tilde{H}_R=(H_{R_1},H_{R_2},H_{R_3};H'_{R_1},H_{R_2},H'_{R_3}),\tilde{c}_p=(c_{p1},c_{p2},c_{p3};c'_{p1},c_{p2},c'_{p3}),\tilde{C}_d=(C_{d_1},C_{d_2},C_{d3};C_{d_1}',C_{d_2},C_{d_2}')$.

Hence, A_{tc} in the IF sense is:

<u>**Case-1**</u>

$$= R_1+(c_{p1},c_{p2},c_{p3};c'_{p1},c_{p2},c'_{p3})\otimes I_{PR}\frac{(a_1+a_2\mu')}{\lambda'(1-\lambda' T_{sp})}\{(T_{sp}-M_T-\frac{\lambda' T_{sp}^{\ 2}}{2}+\frac{\lambda' M_T^{\ 2}}{2})-(1-\lambda' T_{sp})(T_{sp}+M_T)\}$$

$$-S_p I_{ER}[\frac{a_1 M_T^{\ 2}}{2}+\frac{a_2 M_T^{\ 3}}{6}]] \tag{23}$$

After defuzzification of Eq. (23), using the formula mentioned in Eq. (2), we get

$$A_{tc} = R_2+I_{PR}\frac{1}{2}(c_{p2}+\frac{1}{4}(c_{p1}+c'_{p1}+c_{p3}+c'_{p3}))\frac{(a_1+a_2\mu')}{\lambda'(1-\lambda' T_{sp})}\{(T_{sp}-M_T-\frac{\lambda' T_{sp}^{\ 2}}{2}+\frac{\lambda' M_T^{\ 2}}{2})-(1-\lambda' T_{sp})(T_{sp}+M_T)\}$$

$$-S_p I_{ER}[\frac{a_1 M_T^{\ 2}}{2}+\frac{a_2 M_T^{\ 3}}{6}]]] \tag{24}$$

<u>**Case-2**</u>

$$= R_1-S_p I_{ER}[\frac{a_1 M_T^{\ 2}}{2}+\frac{a_2 M_T^{\ 3}}{6}]] \tag{25}$$

After defuzzification of Eq. (25), using the formula mentioned in Eq. (2), we get

$$A_{tc} = R_2-S_p I_{ER}[\frac{a_1 M_T^{\ 2}}{2}+\frac{a_2 M_T^{\ 3}}{6}]] \tag{26}$$

$$R_1 = \frac{1}{T_{sp}}[P_{oc}+(c_{p1},c_{p2},c_{p3};c'_{p1},c_{p2},c'_{p3})\otimes C_w+(H_{r_1},H_{r_2},H_{r_3};H'_{r_1},H_{r_2},H'_{r_3})\otimes[(C-C_w)(\mu'-\frac{\alpha'\mu'^2}{2})-\frac{a_1}{2}\mu'^{\,2}]$$

$$+(H_{o_1},H_{o_2},H_{o_3};H'_{o_1},H_{o_2},H'_{o_3})\otimes[C_w\mu'+\frac{(a_1+a_2\mu')}{1-\lambda' T_{sp}}\frac{(T_{sp}-\mu')^2}{2}]+(C_{d_1},C_{d_2},C_{d3};C_{d_1}',C_{d_2},C_{d_2}')\otimes[(C-C_w$$

$$)\alpha'(\mu'-\frac{\alpha'\mu'^2}{2})-\frac{a_1\alpha'\mu'^{\,2}}{2}+\lambda'[C_w\mu'+\frac{(a_1+a_2\mu')}{1-\lambda' T_{sp}}\frac{(T_{sp}-\mu')^2}{2}]-\mu')^2]+T_N[t_m+(2d_tF_pC_{fc}+d_tF_pS_{fc}w_pO_{rq})+$$

$$(2d_tc_{e1}+d_tc_{e2}O_{rq})(1-\xi(1-e^{-\chi g_c}))]+g_cT_{sp}$$

$$R_2 = \frac{1}{T_{sp}}[P_{oc}+\frac{C_w}{2}(c_{p2}+\frac{1}{4}(c_{p1}+c'_{p1}+c_{p3}+c'_{p3}))+\frac{1}{2}(H_{r_2}+\frac{1}{4}(H_{r_1}+H'_{r_1}+H_{r_3}+H'_{r_3}))[(C-C_w)(\mu'-\frac{\alpha'\mu'^2}{2})-$$

$$\frac{a_1}{2}\mu'^{\,2}]+\frac{1}{2}(H_{o_2}+\frac{1}{4}(H_{o_1}+H'_{o_1}+H_{o_3}+H'_{o_3}))[C_w\mu'+\frac{(a_1+a_2\mu')}{1-\lambda' T_{sp}}\frac{(T_{sp}-\mu')^2}{2}]+\frac{1}{2}(C_{d_2}+\frac{1}{4}(C_{d_1}+C'_{d1}+C_{d_3}+C'_{d_3}$$

$$))[(C-C_w)\alpha'(\mu'-\frac{\alpha'\mu'^2}{2})-\frac{a_1\alpha'\mu'^{\,2}}{2}+\lambda'[C_w\mu'+\frac{(a_1+a_2\mu')}{1-\lambda' T_{sp}}\frac{(T_{sp}-\mu')^2}{2}]]+T_N[t_m+(2d_tF_pC_{fc}+d_tF_pS_{fc}w_pO_{rq})+$$

$$(2d_tc_{e1}+d_tc_{e2}O_{rq})(1-\xi(1-e^{-\chi g_c}))]+g_cT_{sp}$$

5. Optimality Condition:

"The objective of the present model problems is to obtain the minimum value of $A_{tc} / \tilde{A}_{tc}$, which is obtained by solving the following two equations simultaneously, depending on the optimum values of T_{sp} and g_c." The optimality criteria for the minimum total cost:
Necessary and sufficient conditions for optimization for Case 1 and Case 2:

$$\frac{\partial A_{tc}}{\partial T_{sp}} = 0 \ , \ \frac{\partial A_{tc}}{\partial g_c} = 0 \quad \text{(Necessary condition)} \qquad (27)$$

We use the second partial derivatives of $A_{tc} / \tilde{A}_{tc}$ with respect to T_{sp} and g_c verify the positive definiteness of the Hessian matrix H, using the values of the decision variables obtained from Eqs (A.1)-(A.2) and (A.6) (see Appendix A), depending on the cases:

$$H = \begin{pmatrix} \frac{\partial^2 A_{tc}}{\partial T_{sp}^2} & \frac{\partial^2 A_{tc}}{\partial T_{sp} \partial g_c} \\ \frac{\partial^2 A_{tc}}{\partial g_c \partial T_{sp}} & \frac{\partial^2 A_{tc}}{\partial g_c^2} \end{pmatrix}$$

(A)
Eqs (A.3) -(A.5) and (A.7) are highly non-linear (see Appendix A) with more than one independent variable. Due to the highly non-linear expressions, we adopt the numerical approach to obtain the optimal result of the average total cost.

6. Numerical Example

The following parameters are used to obtain the optimum results of g_c, T_{sp}, and $A_{tc} / \tilde{A}_{tc}$:
Parameters used for both models: $I_{PR} = 0.15, M_T = 22$ (for case 1) and $M_T = 35$ (for case 2), $P_{oc} = 600, c_p = 35$, $a_1 = 70, a_2 = 0.5, H_o = 1, H_r = 2, \lambda' = 0.01, \alpha' = 0.04, T_n = 2, C_w = 125, t_m = 0.1, C_{fc} = 0.55, S_{fc} = 1.4, d_t = 100$, $F_p = 0.1, w_p = 4, c_{e1} = 2.35, c_{e2} = 1.3, C_d = 0.5, \xi = 0.5, \chi = 0.7, \mu' = 19, S_p = 50, I_{ER} = 0.12$.
Parameters for IF model:
$c_p = (30,35,40;25,45), H_r = (3,4,5:2,6), H_o = (1.6,1.7,1.8;1.5,1.9)$.
The following is the algorithm for solving the model problem.

6.1 Algorithm (Sahoo, Acharya, and Patnaik [17])

S I, I=1 to 7, indicates the different steps of the algorithm.
S 1: Input all the required parameters.
S 2: Put $g_c = 1$ and any value of $T_{sp} > 0$.
S 3: Compute the value of A_{tc} using equation (22).
S 4: Use $g_c = g_c + 0.1$ to compute A_{tc} given in S 3 until the minimum A_{tc} is reached.
S 5: Save the minimum value of g_c and A_{tc} obtained from S 4 and go to S 6.
S 6: Introduce $T_{sp} = T_{sp} + 0.1$ in equations (22) to compute A_{tc} successively till the minimum value of A_{tc} is not reached.
S 7: Save the values of g_c, T_{sp} and A_{tc}.
Note: IF model also follow the algorithm described above to reach the optimum values.

Table 3.	Optimum results			
	Case 1		Case 2	
DV/Model type for different Cases	Crisp	IF	Crisp	IF
g_c	13	13.2	14.3	14.3
T_{sp}	60.1	51.1	24.2	23.9
Total Avg. cost $(A_{tc} / \tilde{A}_{tc})$	34963.845	38931.893	11485.522	7762.875

6.2 Graphical Illustration

This section includes the graphical illustrations of the optimum results obtained for all the models (shown in Table 3) under consideration for different cases (Figure 1-2 and Figure 3-4 represent the optimum results of DVs for crisp and IF model respectively).

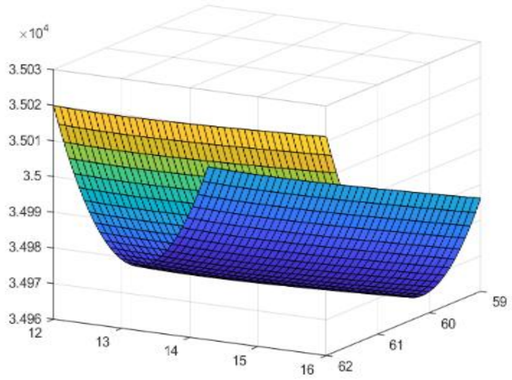

Figure 1 g_c *vs.* T_{sp} *vs.* A_{tc} *for case 1 crisp model*

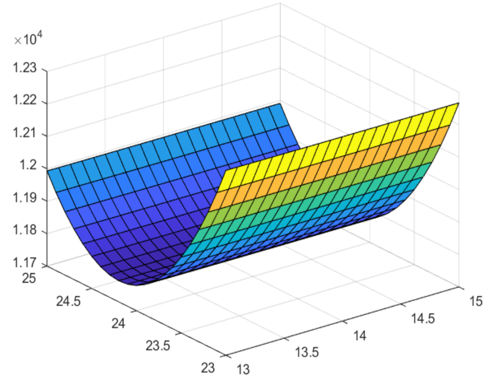

Figure 2 g_c *vs.* T_{sp} *vs.* A_{tc} *for case 2 crisp model*

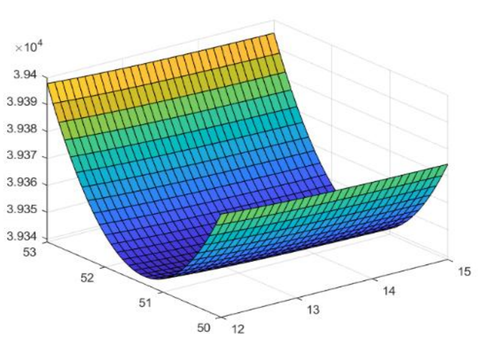

Figure 3 g_c *vs.* T_{sp} *vs.* $\tilde{A}_{tc}$ *for case 1 IF model*

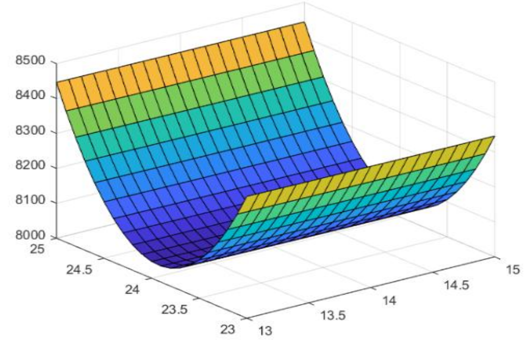

Figure 4 g_c *vs.* T_{sp} *vs.* $\tilde{A}_{tc}$ *for case 2 IF model*

7. Sensitivity investigation and managerial insights: (for both models)

Table 4. Crisp Model

Case 1				Case 2		
a_1	g_c	T_{sp}	$\tilde{A}_{tc}$	g_c	T_{sp}	$\tilde{A}_{tc}$
63	12.8	59.6	31904.420	14.1	24.2	10181.002
66.5	12.9	59.9	33435.036	14.2	24.2	10833.263
73.5	13	60.4	36491.032	14.3	24.2	12137.775
77	13	60.5	38016.809	14.4	24.2	12790.026
a_2	g_c	T_{sp}	$\tilde{A}_{tc}$	g_c	T_{sp}	$\tilde{A}_{tc}$
0.45	13	60.6	34584.271	14.3	24.2	11800.364
0.475	13	60.4	34774.981	14.3	24.2	11642.943
0.525	13	59.9	35150.815	14.3	24.2	11485.522
0.55	13	59.7	35336	14.1	24.2	11359.457
d_t	g_c	T_{sp}	$\tilde{A}_{tc}$	g_c	T_{sp}	$\tilde{A}_{tc}$
90	12.8	58.9	32658.873	14.1	23.8	9586.235
95	12.9	59.5	33817.423	14.2	24	10123.478
105	13	60.7	36098.921	14.4	24.3	12450.256
110	13.1	61.3	37223.439	14.4	24.5	13248.234
I_{ER}	g_c	T_{sp}	$\tilde{A}_{tc}$	g_c	T_{sp}	$\tilde{A}_{tc}$
0.108	13	60.2	35141.562	14.3	24.4	12978.256
0.114	13	60.2	35052.721	14.3	24.4	12589.235
0.126	13	60	34874.856	14.3	23.8	11356.159
0.132	13	60	34785.853	14.3	23.8	11279.357
I_{PR}	g_c	T_{sp}	$\tilde{A}_{tc}$	g_c	T_{sp}	$\tilde{A}_{tc}$

0.54	13	60.5	34644.339	14.3	24.5	10852.729
0.57	13	60.3	34804.592	14.3	24.3	11385.258
0.63	13	60	35122.064	14.3	23.8	12478.589
0.66	13	59.8	35279.247	14.3	23.6	12989.456
F_p	g_c	T_{sp}	$\tilde{A}_{tc}$	g_c	T_{sp}	$\tilde{A}_{tc}$
0.09	13	59.6	33903.512	14.3	23.4	11158.123
0.095	13	59.9	34434.936	14.3	23.7	11345.231
0.105	13	60.4	35490.313	14.3	24.5	11653.423
0.11	13	60.7	36014.464	14.3	24.9	11985.789

Table 5 IF Model

Case 1				Case 2		
a_1	g_c	T_{sp}	$\tilde{A}_{tc}$	g_c	T_{sp}	$\tilde{A}_{tc}$
63	13	50.5	35592.953	14.1	23.3	7509.890
66.5	13.1	50.8	37264.536	14.2	23.6	7615.589
73.5	13.3	51.4	40595.490	14.4	24.1	7923.564
77	13.3	51.8	42256.099	14.4	24.4	8102.741
a_2	g_c	T_{sp}	$\tilde{A}_{tc}$	g_c	T_{sp}	$\tilde{A}_{tc}$
0.45	13.2	51.8	38431.702	14.3	24.3	7482.302
0.475	13.2	51.4	38683.943	14.3	24.1	7512.012
0.525	13.2	50.8	39175.705	14.3	23.7	7812.230
0.55	13.2	50.5	39415.444	14.3	23.5	8215.230
d_t	g_c	T_{sp}	$\tilde{A}_{tc}$	g_c	T_{sp}	$\tilde{A}_{tc}$
90	13	49.7	36212.102	14.1	22.5	7399.150
95	13.1	50.4	37581.256	14.2	23.2	7530.521
105	13.3	51.8	40265.363	14.4	24.5	7945.105
110	13.3	52.4	41582.674	14.4	25.1	8259.483
I_{ER}	g_c	T_{sp}	$\tilde{A}_{tc}$	g_c	T_{sp}	$\tilde{A}_{tc}$
0.108	13.2	51.2	39140.931	14.3	24	8089.569
0.114	13.2	51.2	39036.474	14.3	24	7815.018
0.126	13.2	51.1	38827.232	14.3	23.8	7523.569
0.132	13.2	51.1	38722.436	14.3	23.8	7398.235
I_{PR}	g_c	T_{sp}	$\tilde{A}_{tc}$	g_c	T_{sp}	$\tilde{A}_{tc}$
0.54	13.2	51.7	38490.751	14.3	24.5	7499.791
0.57	13.2	51.4	38712.748	14.3	24.2	7601.095
0.63	13.2	50.9	39148.213	14.3	23.5	7984.705
0.66	13.2	50.6	39361.627	14.3	23.1	8285.010
F_p	g_c	T_{sp}	$\tilde{A}_{tc}$	g_c	T_{sp}	$\tilde{A}_{tc}$
0.09	13.2	50.5	37682.768	14.3	23.4	7409.369
0.095	13.2	50.8	38309.233	14.3	23.7	7569.589
0.105	13.2	51.4	39550.864	14.3	24.2	7854.874
0.11	13.2	51.7	40166.259	14.3	24.6	7912.298

Sensitivity Investigation on models with crisp and IF considerations is analyzed below. This study is very relevant to understanding the business scenario for the item. The effect of changes took place for the DV under the consideration of the percentage change in parameters (taken one parameter at a time) on g_c, T_{sp} and $A_{tc} / \tilde{A}_{tc}$,based upon these changes, the following decisions may be adopted by the DM.:

- As $A_{tc} / \tilde{A}_{tc}$ gets a similar impact with the change in d_t therefore, the DM may proceed with the shortest distance factors for supplying items. Similar considerations can also be implemented for F_p .
- Again, when the per-unit cost of c_p increases, $A_{tc} / \tilde{A}_{tc}$ also increases. Hence, the DM should have proper planning for the number of cycles, the cycle time for the rented warehouse (μ') , and the units to be purchased.
- We also observe that $A_{tc} / \tilde{A}_{tc}$ decreases when I_{ER} increases, implying that more benefits can be found with the proper use of the permissible delay in payment option for the retailer. Therefore, the retailer should plan to order fewer than the usual amount more frequently to get the benefits of permissible delay in payment.

• As a_1 and a_2 increase, either O_{rq} increases or remains constant, whereas $A_{tc} / \tilde{A}_{tc}$ increases. Hence a_1 and a_2 have the most substantial impact on the business, which means that when retailers have a larger share of the market, there will be more demand. From Tables 4 and 5, it can be observed that with higher demand, either g_c increases or remains constant, whereas, $A_{tc} / \tilde{A}_{tc}$ increases. Hence when a_1 and a_2 increase, the rate of change of $A_{tc} / \tilde{A}_{tc}$ reduces, leading to the proposed model's market stability.

• Again, for the proposed model I_{PR} increases, $A_{tc} / \tilde{A}_{tc}$ increases. This emphasizes that the retailer should settle all payments within the time stipulated by the supplier to reduce the total cost.

8. Conclusion

This study develops a two-warehouse model for retailers with optimum green investment and single-trade credit options. The model emphasizes efficient transportation from OWH to RWH and ultimately to the customer, minimizing carbon emissions and the total cost. The research aims to optimize the cost of GT, the total cycle length, and the total average cost for crisp and IF models. Because of the fuzziness in per unit purchasing, holding, and disposal costs, which are not known precisely, we consider IF numbers and defuzzify the total cost by α, β - cut method. We validate the model problem by taking appropriate numerical examples. This research can be extended by including carbon emissions associated with holding the items in RWH and OWH. Also, the extension can include the fuzziness of the scale parameters in the demand function and the rate of deterioration.

Appendix A

Case-1

$$\frac{\partial A_{tc}}{\partial T_{sp}} = A_1 + A_2 + c_p I_{PR}\left(\frac{a_1 + a_2\mu'}{\lambda'}\right)\{(\frac{1}{1-\lambda' T_{sp}})(T_{sp} - M_T - \frac{\lambda' T_{sp}^2}{2} + \frac{\lambda' M_T^2}{2}) - T_{sp} + M_T\} - S_p I_{ER}[\frac{a_1 M_T^2}{2} + \frac{a_2 M_T^3}{6}]]$$
$$+\frac{1}{T_{sp}}[H_o \frac{a_1 + a_2\mu'}{(1-\lambda' T_{sp})^2}\{-\frac{T_{sp}^2}{2} + T_{sp} - \frac{\lambda' T_{sp}^2}{2} + \frac{\mu'^2\lambda'}{2} - \mu'\} + C_d \frac{\lambda'(a_1 + a_2\mu')}{2(1-\lambda' T_{sp})}\{2T_{sp} - 2\mu' - 3\lambda' T_{sp}^2 + \mu'^2\lambda'\} + g_c +$$
$$c_p I_{PR}\left(\frac{a_1 + a_2\mu'}{\lambda'(1-\lambda' T_{sp})^2}\right)\{1 - \lambda' M_T + \frac{T_{sp}^2}{2} - T_{sp} + \frac{\lambda' T_{sp}^2}{2} + \frac{\lambda' M_T^2}{2} - \frac{1}{(1-\lambda' T_{sp})^2}\}]$$

(A.1)

$$\frac{\partial A_{tc}}{\partial g_c} = \frac{T_N}{T_{sp}}[-d_t c_{e2} C\chi\xi e^{-\chi g_c}] + 1, \text{ Where } \lambda' T_{sp} \neq 1$$

(A.2)

$$\frac{\partial^2 A_{tc}}{\partial T_{sp}^2} = A_3 + \frac{2c_p I_{PR}}{T_{sp}^3}\left(\frac{a_1 + a_2\mu'}{\lambda'(1-\lambda' T_{sp})^2}\right)\{1 - \lambda' M_T + \frac{T_{sp}^2}{2} - T_{sp} + \frac{\lambda' T_{sp}^2}{2} + \frac{\lambda' M_T^2}{2} - \frac{1}{(1-\lambda' T_{sp})^2}\}] + \frac{1}{T_{sp}}[H_o$$
$$\frac{a_1 + a_2\mu'}{(1-\lambda' T_{sp})^3}\{-T_{sp} + (1+\lambda' T_{sp})^2 - \lambda' T_{sp} + \mu'^2\lambda'^2 - \mu'\lambda'\} + C_d \frac{\lambda'(a_1 + a_2\mu')}{2(1-\lambda' T_{sp})^2}\{2 - 2\mu'\lambda' - 3\lambda'(1-\lambda' T_{sp}^2) +$$
$$\mu'^2\lambda'^2\} + c_p I_{PR}\left(\frac{a_1 + a_2\mu'}{\lambda'(1-\lambda' T_{sp})^3}\right)\{(1 - \lambda' M_T + \frac{\lambda' M_T^2}{2})2\lambda' + T_{sp}(1+\lambda') - \frac{4\lambda'}{(1-\lambda' T_{sp})^2}\}]$$

(A.3) $$\frac{\partial^2 A_{tc}}{\partial g_c^2} = \frac{T_N}{T_{sp}}[d_t c_{e2} C_w \chi^2 \xi e^{-\chi g_c}]$$

(A.4) $$\frac{\partial^2 A_{tc}}{\partial g_c \partial T_{sp}} = \frac{T_N}{T_{sp}^2}[d_t c_{e2} C\chi\xi e^{-\chi g_c}]$$

(A.5)

Case-2

$$\frac{\partial A_{tc}}{\partial T_{sp}} = A_1 + A_2$$

(A.6)

$$\frac{\partial^2 A_{tc}}{\partial T_{sp}^2} = A_3$$

(A.7)

Where

$$A_1 + A_2 = -\frac{1}{T_{sp}^{\ 2}}[P_{oc} + c_p C + H_r[(C - C_w)(\mu' - \frac{\alpha'\mu'^2}{2}) - \frac{a_1}{2}\mu'^2] + H_o C_w \mu' + H_o(\frac{a_1 + a_2\mu'}{1-\lambda' T_{sp}})[\frac{(T_{sp}-\mu')^2}{2}] + C_d[(C - C_w)(\alpha'\mu' - \frac{\alpha'^2\mu'^2}{2}) - \frac{a_1\alpha'\mu'^2}{2} + C_w\lambda'\mu's + \frac{\lambda'(a_1 + a_2\mu')}{2(1-\lambda' T_{sp})}(T_{sp}-\mu')^2] + T_N[t_m + (2d_t F_p C_{fc} + d_t F_p S_{fc} w_p O_{rq}) + (2d_t c_{e1} + d_t c_{e2} O_{rq})(1 - \xi(1 - e^{-\chi g_c}))] + g_c T_{sp} - S_p I_{ER}[\frac{a_1 M_T^{\ 2}}{2} + \frac{a_2 M_T^{\ 3}}{6}]] + \frac{1}{T_{sp}}[H_o \frac{a_1 + a_2\mu'}{(1-\lambda' T_{sp})^2}\{-\frac{T_{sp}^{\ 2}}{2} + T_{sp} - \frac{\lambda' T_{sp}^{\ 2}}{2} + \frac{\mu'^2\lambda'}{2} - \mu'\} + C_d \frac{\lambda'(a_1 + a_2\mu')}{2(1-\lambda' T_{sp})}\{2T_{sp} - 2\mu' - 3\lambda' T_{sp}^{\ 2} + \mu'^2\lambda'\} + g_c]$$

$$A_3 = \frac{2}{T_{sp}^{\ 3}}[P_{oc} + c_p C + H_r[(C - C_w)(\mu' - \frac{\alpha'\mu'^2}{2}) - \frac{a_1}{2}\mu'^2] + H_o C_w \mu' + H_o(\frac{a_1 + a_2\mu'}{1-\lambda' T_{sp}})[\frac{(T_{sp}-\mu')^2}{2}] + C_d[(C - C_w)(\alpha'\mu' - \frac{\alpha'^2\mu'^2}{2}) - \frac{a_1\alpha'\mu'^2}{2} + C_w\lambda'\mu's + \frac{\lambda'(a_1 + a_2\mu')}{2(1-\lambda' T_{sp})}(T_{sp}-\mu')^2] + T_N[t_m + (2d_t F_p C_{fc} + d_t F_p S_{fc} w_p C) + (2d_t c_{e1} + d_t c_{e2} O_{rq})(1 - \xi(1 - e^{-\chi g_c}))] + g_c T_{sp} - S_p I_{ER}[\frac{a_1 M_T^{\ 2}}{2} + \frac{a_2 M_T^{\ 3}}{6}]] - \frac{1}{T_{sp}^2}[\frac{(a_1 + a_2\mu')(2T_{sp} - 2\mu' - T_{sp}^2\lambda')}{2(1-\lambda' T_{sp})^2}(H_o + C_d\lambda') + g_c] - T_{sp}^2[H_o \frac{a_1 + a_2\mu'}{(1-\lambda' T_{sp})^2}\{-\frac{T_{sp}^{\ 2}}{2} + T_{sp} - \frac{\lambda' T_{sp}^{\ 2}}{2} + \frac{\mu'^2\lambda'}{2} - \mu'\} + C_d \frac{\lambda'(a_1 + a_2\mu')}{2(1-\lambda' T_{sp})}\{2T_{sp} - 2\mu' - 3\lambda' T_{sp}^{\ 2} + \mu'^2\lambda'\} + g_c] + \frac{1}{T_{sp}}[H_o \frac{a_1 + a_2\mu'}{(1-\lambda' T_{sp})^3}\{-T_{sp} + (1 + \lambda' T_{sp})^2 - \lambda' T_{sp} + \mu'^2\lambda'^2 - \mu'\lambda'\} + C_d \frac{\lambda'(a_1 + a_2\mu')}{2(1-\lambda' T_{sp})^2}\{2 - 2\mu'\lambda' - 3\lambda'(1 - \lambda' T_{sp}^2) + \mu'^2\lambda'^2\}]$$

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