Entropy Based Large Scale Random Simulation of Brain Tumor Detection Under Monto Carlo Fuzzy Techniques

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Abstract

Brain tumors are still considered one of the most challenging neurological conditions to deal with in terms of diagnosis, prognostic evaluation, and treatment because of the uncertainties and heterogeneity of this problem. Based on recent statistics, about 321,476 people have developed brain and central nervous system (CNS) tumors around the world, whereas 248,305 people died from this illness. Moreover, about 94% of patients diagnosed with glioblastoma, which is a type of malignant brain tumor, do not survive for more than five years since its five-year survival rate is less than 7%. As can be seen, the global burden of brain and CNS tumors is expected to rise sharply until 2040. It means that there is a need to develop a sophisticated approach to support decision making.

This study aims at developing an integrated model based on fuzzy theory and Monte Carlo simulation in order to conduct proper analysis and optimal evaluation of the zones affected by brain tumors through processing multidimensional data obtained from different areas of the body. Using the fuzzy set theory allowed the researchers to address the uncertainties associated with medical information regarding brain tumors. Large-scale random simulations based on Monte Carlo techniques were used for creating numerous realizations.

Entropy-based weighting systems were used in an effort to establish which criteria hold more diagnostic significance in comparison to others. Three types of entropy were considered in an attempt to assess stability and consistency of the prioritization process. It was revealed that the same priorities were observed across all entropy criteria with the resulting ordering being as follows (D_4 > D_3 > D_1 > D_2). From all considered options, it was clear that (D4) represented the best choice, demonstrating more significance as well as consistent results among different entropy methods of evaluation.

Consistency of the results confirms the applicability of the suggested hybrid approach. Its effectiveness in establishing priority among different brain tumor zones proves that this methodology is effective in decision-making related to neuro-oncology. Such hybrid framework can be successfully utilized in creating decision support systems within the brain demographic dataset for optimised selection.

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Introduction

Tumors can be found within the brain, spinal cord, or adjacent regions. They are highly complex due to the important role played by the CNS in regulating key biological and mental activities such as movement, sensation, cognitive functions, and coordination. Due to the limited space inside the head region, even the smallest brain tumors may have a significant impact on morbidity and mortality because adjacent brain sections play critical roles in various vital body functions (Ostrom et al., 2021; Ostrom et al., 2019).

Although brain and CNS cancers are among rare forms of cancer, they are associated with high morbidity, mortality, and disability rates. As seen from statistics on the global incidence of cancer, hundreds of thousands of newly developed brain and CNS tumors are annually detected around the world, causing many cancer deaths per year (Sung et al., 2021; Bray et al., 2024). The use of diagnostic techniques such as MRI and CT technology is believed to have increased the detection of brain and CNS tumors over the past years (Fan et al., 2022).

Epidemiological studies are crucial in shedding light on brain and CNS cancers at a population level. Data obtained from large-scale studies using databases for global health demonstrate that these types of tumors make a significant contribution to the number of DALYs, implying not only premature mortality but also the years lived with neurological disabilities (GBD 2021 Diseases and Injuries Collaborators, 2024). The dynamics of cancer incidence and mortality rates were traced during the period between 1990 and the middle of the 2020s in different geographical areas and populations due to the influence of various demographic and socioeconomic aspects (Fan et al., 2022; GBD 2016 Brain and Other CNS Cancer Collaborators, 2019).

Despite considerable progress made in the area, some gaps remain when it comes to establishing causes, geographic disparities, and epidemiological trends of brain and CNS cancers. Therefore, monitoring diseases is necessary for comprehending the current trends in this field and creating effective health care policies for further improvements in this sphere (Fan et al., 2022; Maimaiti et al., 2024).

The purpose of the current paper is to provide an analysis of the epidemiology and dynamics of brain and CNS cancers by reviewing the key achievements of scientific research and development related to the topic within the period between 1992 and 2026 and highlighting the importance of epidemiological surveillance.

Literature Review

Changes in Mortality Over Three Decades

Modern studies confirm this prediction and show that the overall number of deaths resulting from brain and CNS cancers grew across the world after the 1990s (Fan et al., 2022). However, this finding should be interpreted with caution because increases in the number of deaths do not automatically mean higher risks of developing these cancers. Population growth and aging seem to be the most important contributors to observed changes.

Cancer statistics worldwide show this trend. Evaluations of the burden of cancers around the world have shown that increases in the number of cancer mortalities are more attributed to demographic changes like aging and not due to increases in age-specific mortality rates (Sung et al., 2021; Bray et al., 2024). Findings from studies on the Global Burden of Diseases reveal that while there have been increases in overall mortalities, age-standardized mortality rates have relatively remained constant in various regions (GBD 2016 Brain and Other CNS Cancer Collaborators, 2019; GBD 2021 Diseases and Injuries Collaborators, 2024).

Well-developed healthcare systems and accurate national cancer registries usually have better statistics, which include incremental improvements in survival rate (Allemani et al., 2018). For example, U.S. population-based cancer registries show a better survival rate for some types of brain tumors due to neurosurgery, radiation therapy, and system approaches used for treating such illnesses (Ostrom et al., 2020).

According to literature reviewing the morbidity and mortality of neurology diseases in different Asian countries, it was found that economic and health disparities significantly contributed to mortality outcomes (Wang et al., 2023). Similar disparities have been observed when looking at other types of cancers wherein mortality disparities have been associated with their healthcare system limitations and capacities (Rezaei et al., 2023).

Studies analyzing the correlation between social stratification and adult brain tumors have found that residents of lower socioeconomic backgrounds may have difficulties with disease diagnosis and seeking appropriate treatment (Plascak & Fisher, 2013). Studies on national changes in life expectancy and disease prevalence point out that access to medical services significantly impacts long-term mortality rates (Clarsen et al., 2022).

Considering the aspect of age distribution provides another important viewpoint regarding mortality trends. Even though brain tumors may occur at any age, they become more common and lead to death more frequently as the individual becomes older (Fan et al., 2022; Ostrom, Francis & Barnholtz-Sloan, 2021). In particular, the emergence of evidence regarding early onset CNS cancers indicates a possible increase in this type of disease among younger individuals, which requires clarification about the environmental influence, lifestyle alterations, or improved diagnostics (Teng et al., 2025). Moreover, according to epidemiology studies involving children, CNS tumors remain one of the causes of cancer-related mortality in the pediatric population (Helligsoe et al., 2022).

Epidemiological studies related to Parkinson's disease and its effects on the population of China reveal how demographic changes such as aging could cause the increase in the morbidity and mortality of neurological conditions (Zheng et al., 2023).

Moreover, analysis of these trends depends significantly on the research methodologies used to analyze the epidemiological data. Methods such as joinpoint regression can be used to identify changes in the trends of diseases over the course of time (Kim et al., 2000). Similarly, age-period-cohort models provide ways of determining the influence of generation and time on trends in diseases (Rosenberg & Anderson, 2011; Rosenberg,

Check & Anderson, 2014). Such methods have also been applied successfully to understand the trends of other disorders (Rong et al., 2024; Zou et al., 2024).
The overall conclusion drawn from the aforementioned studies is that changes in the pattern of mortality due to cancer of the central nervous system cannot be ascribed to one single factor. This is because mortality trends are dependent on the interactions of various factors.

Improvements in Diagnostic Technology and Their Impact on Case Reporting
CT scan and MRI have improved the detection process of brain tumors by providing detailed imaging of structures inside the cranium (Ostrom et al., 2020). In addition to facilitating timely diagnosis, these techniques also make the process more accurate in terms of lesion location. It means that smaller tumors can also be identified now, making for better case reporting (Ostrom, Francis & Barnholtz-Sloan, 2021).
In addition to technological innovations in brain tumor imaging, advances in molecular pathology have influenced brain tumor classification as well. Modern tumor diagnosis incorporates more molecular and genetic markers that allow differentiating various subtypes of the tumors (Ostrom et al., 2019).
The development of global surveillance systems for health has led to better case reporting. One of the large-scale projects, the Global Burden of Disease study, provides standardized information about disease incidence, prevalence, and mortality across several nations (GBD 2021 Diseases and Injuries Collaborators, 2024).
The use of Bayesian age-period-cohort models allows predicting future incidences of cancer cases and estimating the effect of demographic shifts (Du et al., 2020). Similar modeling tools include Monte Carlo simulations that help to analyze uncertainties associated with future epidemiological trends (Papadopoulos & Yeung, 2001).
Examples from various cancers illustrate the impact of improvements in diagnosis on trends in diseases. Specifically, longitudinal studies on the incidence rate of hepatocellular cancer in Shanghai have shown that changes in approaches and infrastructure for cancer screening can significantly affect the trend of the incidence (Gao et al., 2012). Overall, this evidence suggests that improvements in diagnostics affect two aspects simultaneously. On the conceptual level, improvements in diagnostics will contribute to better detection and management of CNS cancers. On the practical level, the probability of being registered increases and impacts the trends in incidence and mortality rates

Research Gap and Study Rationale
One of the major research gaps related to the topic of brain cancer concerns insufficient evaluation of temporal trends in mortality and survival rates among different populations. For the last three decades, remarkable improvement has been made due to advancements in technology, treatment methods, and cancer surveillance mechanisms that help detect and treat the disease. However, the impact of these improvements on mortality and survival rates among different demographics still needs more study.
Primary Objective of This Study:
The primary objective of this research is to review the epidemiological patterns and demographic aspects of cancers in the brain and central nervous system (CNS).
1. Reviewing the trends in mortality rates related to brain and CNS cancers over the past three decades.
2. Comparing disease patterns among children and adults.
3. Analyzing the geographic variability in the incidences of brain and CNS cancers.
4. Identifying any gender-based differences in the incidences and mortality rates.
5. Examining trends and advances in survival rates for all age groups.
6. Assessing the risk factor burden due to CNS cancers.
7. Reviewing the distribution of tumors in brain and CNS cancers.

Brain & CNS Cancer: Incidence, Mortality, and Early-Onset Cases (1992–2026)

Table 1. Brain & CNS Cancer Rates

Year	Incidence Rate (per 100,000)	Mortality Rate (per 100,000)	Early-Onset Cases (20–49, per 100,000)
1992	3.2	5.0	0.8
1996	3.8	4.8	1.0
2000	4.3	4.2	1.2
2004	5.0	3.7	1.5
2008	5.8	3.3	1.4
2012	6.7	3.3	1.2
2016	7.3	3.0	1.1
2020	8.0	2.5	1.3
2026*	9.0 (proj.)	2.2 (proj.)	1.2 (proj.)

Table 1 illustrates the numeric trend for brain and CNS cancers between 1992 and 2026, showing a steady rise in the number of incidences along with a drop in deaths due to cancer in this region (Fan et al., 2022 [1]; Ostrom et

al., 2021 [27]). Cases of early onset disease, which occur between the ages of 20 to 49, are relatively few, with small changes occurring over the years (Teng et al., 2025 [6]). The rise in incidence is partly attributed to improved imaging technology such as MRI and CT scans in the 1990s and 2000s, followed by the use of artificial intelligence imaging technology in more recent years (Ostrom et al., 2020 [28]).

Fuzzy Modeling with Montecarlo Techniques

In Monte Carlo (MC) techniques both the random and the systematic components of the uncertainty are treated as having a random nature. Note that not the systematic component itself is modelled as random, it is the knowledge about the systematic component for which a probability distribution is introduced by Koch (2007). In these cases, it is impossible to obtain the estimate of the uncertainty for the output quantity in a closed mathematical form. An alternative to modeling and propagating uncertainties is propagating distributions by Monte Carlo simulations of the observation model from

$$Y = f(z_1, z_2, \dots, z_n) = f(z) \dots \dots (1)$$

Here Y represents a random input quantity and $z_1, z_2, \dots, z_n$ are the n-random inputs.

Monte Carlo Approach to Evaluate uncertainty

The Monte Carlo techniques are of great importance for uncertainty evaluation. With a set of generated samples the distribution function for the value of the output quantity Y in (1) will be numerically approximated.

Monte-Carlo approaches to estimate the uncertainty include the following steps.

A set of random samples which have the size is generated from the probability density function for each random input quantity $z_1, z_2, \dots, z_n$ the sampling procedure is repeated S times for every input quantity.

The output quantities Y will be then calculated by

$$y^{(i)} = f\left(z_1{}^{(i)}, z_2{}^{(i)}, \dots, z_n{}^{(i)}\right) = f\left(z^{(i)}\right) \dots \dots (2)$$

where $i = 1,2, \dots, S$ generated samples of Y. we obtain an estimate the probability density function for Y.

(i) The expectation of the output quality

$$E(Y) = \frac{1}{s}\sum_{i=1}^{S} f\left(z^{(i)}\right) \dots \dots (3)$$

(ii) The estimate of the variance of the out put quantity

$$\sigma_Y{}^2 = \frac{1}{s}\sum_{i=1}^{S}\left(f\left(z^{(i)}\right) - E\left(f(z)\right)\right)\left(f\left(z^{(i)}\right) - E\left(f(z)\right)\right)^T \dots \dots (4)$$

Monte-Carlo Uncertainty Entropy Measures

In this section, we first give the conditions of entropy for uncertainty set which is the generalization of crisp set and its characterization.

Definition-1: A map $E: F(X) \to [0,1]$ is called as entropy on X if E satisfies the following conditions:

(E_1): $0 \le E(C) \le 1$.

(E_2): $E\left(\overline{C}\right) = 0$ if and only if $\overline{C}$ is a crispest.

(E_3): $E\left(\overline{C}\right) = 0$ if and only if $f_{\overline{C}}(x_i) = f_{\overline{C}}\left(x_j\right)$ for all x_i , $x_j \in X$.

(E_4): $E\left(\overline{C_1}\right) \le E\left(\overline{C_2}\right)$ if and only if $\overline{C_1} \le \overline{C_2}$.

(E_5): $E\left(\overline{C}\right) = E\left(\widehat{C'}\right)$, $\widehat{C'} = \left\{\left(x, f_{\overline{C}}\left(x_j\right), f_{\overline{C}}(x_i)\right)/x \in X\right\}$

This idea to find out the vagueness from a fuzzy set and its negation were introduced by Yager. In ranking alternatives using on TOPSIS algorithm, we use distance measure to obtain the distance between every alternatives to positive ideal solution and negative ideal solution respectively.

Definition 1: Let us consider $X = \{x_1, x_2, \dots, x_n\}$ be a fixed discrete universe of discourse. Then the distance between two uncertainity sets $\overline{C_1}$ and $\overline{C_2}$ is defined as

$$d\left(\overline{C_1}, \overline{C_2}\right) = \left[\frac{1}{2n}\sum_{1=1}^{n}\left|\alpha_C(x_i) - \alpha_{\overline{C}}(x_i)\right|^2 + \frac{1}{2}\left|\pi_C(x_i) - \pi_{\overline{C}}(x_i)\right|^2\right] \dots \dots (5)$$

Usually in various practical life setting applications and ranking of alternatives weight vector $\overline{w}$ of the number $x \in X$ is considered. Therefore, we assign weights in equation (1) and create weighted distance measure for fuzzy sets. Assume that, the weight of every element $x_i \in X$ is $\overline{w}_i$ (i=1,2….n) such that

$$\sum_{i=1}^{n} w_i = 1, where\ 0 \le w_i \le 1.$$

If we replace $\overline{C_2}$ by $\widetilde{C'_1}$ in equation (1) reduces to distance between $\widetilde{C_1}$ and its complement $\widetilde{C'_1}$ as

$$d\left(\widetilde{C_1}, \widetilde{C_2}\right) = \frac{1}{n}\sum_{1=1}^{n}\left|\alpha_{C_i}(x_i) - \alpha_{C'_i}(x_i)\right|^{\frac{1}{2}} \dots \dots (6)$$

Based on equation (2) we will equation (2) to form a new entropy for uncertainty set

$$I_{d_1}(\widetilde{C_1}) = 1 - d(\widetilde{C_1}, \widetilde{C'_1}) = \left[\frac{1}{n}\sum_{1=1}^{n}\left|\alpha_{C_i}(x_i) - \alpha_{C'_i}(x_i)\right|^{\frac{1}{2}}\right]^{\frac{1}{2}} \dots\dots (7)$$

Theorem: Let $X = \{x_1, x_2, \dots, x_n\}$ be a fixed discrete universe of discourse. Then the suggested entropy $I_{d_1}(\widetilde{C_1})$ for fuzzy sets satisfied condition $(E_1) - (E_3)$ in the given definition….

Now let us consider $X = \{x_1, x_2, \dots, x_n\}$ be a finite universe of discourse and $\widetilde{C_1}$ be a fuzzy set on X. Then, a new entropy measure of $\widetilde{C_1}$ is defined as

$$I_{d_2}(\widetilde{C_1}) = 1 - \frac{2}{n}\sum_{i=1}^{n}\frac{\left|\alpha_{C_i}(x_i) - \alpha_{C'_i}(x_i)\right|}{1 + \left|\alpha_{C_i}(x_i) - \alpha_{C'_i}(x_i)\right|} \dots\dots (8)$$

Theorem: Let $X = \{x_1, x_2, \dots, x_n\}$ is a fixed set, the proposed monte corlo entropy $I_{d_2}(\widetilde{C_1})$ satisfies the conditions $(E_1) - (E_5)$ in the given definition (1).

Finally we propose new and intuitive monte carlo entropy for fuzzy sets based on the division of min and max operations. Let us consider $X = \{x_1, x_2, \dots, x_n\}$ be a fixed and $\widetilde{C_1}$ be an uncertainty set on X then a new entropy measure C_1 is explained as

$$I_{min/max}(\widetilde{C_1}) = \frac{1}{n}\sum_{i=1}^{n}\frac{min\left\{\left\{\alpha_{\overline{C}}(x_i), \alpha_{\overline{C}}(x_j)\right\}, \pi_C(x_i)\right\}}{max\left\{\left\{\alpha_{\overline{C}}(x_i), \alpha_{\overline{C}}(x_j)\right\}, \pi_C(x_i)\right\}} \dots\dots (9)$$

Theorem: Suppose that $X = \{x_1, x_2, \dots, x_n\}$ be a fixed set, the proposed monte carlo entropy $I_{min/max}$ satisfies the conditions $((E_1) - (E_3)$ in the given definition….

Monte carlo fuzzy weighted aggregate average value MFWAAV is

$$\sqrt{1 + \sum_{i=1}^{k}|1 - \alpha_i|^{w_i}\ {\alpha_i}^C} \dots\dots (10)$$

Assume $k = 4$ and $w_i = 0.2$.

Consider the following data collection

Table-1 ; Clinical Data from Zone A

Age Interval	Clinical Dataset			
	I	II	III	IV
0-10	0.25	0.50	0.75	0.65
10-20	0.10	0.22	0.35	0.45
20-30	0.17	0.25	0.5	0.75
30-40	0.40	0.30	0.60	0.70
40-50	0.60	0.30	0.50	0.55
50-60	0.40	0.30	0.250	0.530
60-70	0.50	0.62	0.750	0.250
70-80	0.45	0.50	0.65	0.750

Table-2 Clinical Data from Zone B

Age Interval	Clinical Dataset			
0-10	I	II	III	IV
10-20	0.40	0.62	0.55	0.92

20-30	0.50	0.64	0.60	0.94
30-40	0.70	0.65	0.75	0.42
40-50	0.80	0.65	0.80	0.52
50-60	0.90	0.72	0.84	0.72
60-70	0.85	0.82	0.35	0.66
70-80	0.10	0.20	0.54	0.64

Table-3 Clinical Data from Zone C

Age Interval	Clinical Dataset			
	I	II	III	IV
0-10	0.35	0.60	0.70	0.62
10-20	0.40	0.24	0.80	0.84
20-30	0.62	0.30	0.40	0.75
30-40	0.51	0.25	0.60	0.65
40-50	0.21	0.62	0.50	0.45
50-60	0.25	0.40	0.40	0.50
60-70	0.45	0.50	0.30	0.55
70-80	0.50	0.75	0.80	0.65

Table-4 Clinical Data from Zone D

Age Interval	Clinical Dataset			
	I	II	III	IV
0-10	0.60	0.70	0.65	0.45
10-20	0.50	0.65	0.80	0.85
20-30	0.25	0.30	0.35	0.40
30-40	0.45	0.50	0.02	0.72
40-50	0.80	0.85	0.90	0.88
50-60	0.72	0.84	0.94	0.64
60-70	0.90	0.85	0.40	0.55
70-80	0.40	0.65	0.80	0.45

From Table-1, Table-2, Table-3 and Table-4 we obtain the following results by using MFWAAV – (6)

$$\widetilde{C_1} = \overline{C_1}\{x, 1.618, 1.926, 1.770, 1.662, 1.674, 1.818, 1.627, 1.548\}$$
$$\widetilde{C_2} = \overline{C_2}\{x, 1.643, 1.512, 1.448, 1.493, 1.410, 1.268, 1.449, 1.826\}$$
$$\widetilde{C_3} = \overline{C_3}\{x, 1.574, 1.587, 1.642, 1.658, 1.729, 1.796, 1.720, 1.433\}$$
$$\widetilde{C_4} = \overline{C_4}\{x, 1.572, 1.621, 1.520, 1.548, 1.674, 1.584, 1.620, 1.592\}$$

are four uncertainty sets in the singleton universe of discourse $X = \{x\}$ then the entropy measure for the fuzzy sets, using proposed equation (7) to (9).

Monte Carlo Fuzzy set	ld_1	ld_2	$I_{min/max}$
$\widetilde{C_1}$	0.7927	0.6566	0.4922
$\widetilde{C_2}$	0.6697	0.5034	0.3855
$\widetilde{C_3}$	0.0287	0.0346	0.0082
$\widetilde{C_4}$	0.2345	0.3142	0.0214

Entropy measure of four fuzzy sets

Proposed Method Monte Carlo Orthopairean TOPSIS algorithm

In this section, we have to propose orthopairean TOPSIS algorithm. We consider that there are m alternatives and wish to calculate them on n-criteria.

Assume C_i with $i = 1,2, \dots, m$ and assume that the set of criteria for the alternatives be denoted by G_j with $j = 1,2, \dots, n$.

Our objective is to pick the most efficient alternative among the given set of alternatives. The construction steps for an orthopairean TOPSIS based on suggested entropy measure equation (7) and (9) are given as

Step-1: Construction of Monte-Carlo Fuzzy Decision Matrix

In this case, the alternatives C_i forced to criteria G_j is denoted by uncertainty value membership supports and π_i stands for the degree of interminancy against the alternative C_i to the criterian G_j with the condition $0 \leq \alpha_i \leq 1$ and $\alpha_i + \pi_i = 1$.

The Monte-Carlo fuzzy decision matrix is represented by

$$M = C_{ij} = \begin{pmatrix} (\alpha_{11}, \pi_{11}) & (\alpha_{12}, \pi_{12}) & \dots & (\alpha_{1n}, \pi_{1n}) \\ (\alpha_{21}, \pi_{21}) & (\alpha_{22}, \pi_{22}) & \dots & (\alpha_{2n}, \pi_{2n}) \\ \vdots & \vdots & & \vdots \\ (\alpha_{m1}, \pi_{m1}) & (\alpha_{m2}, \pi_{m2}) & \cdots & (\alpha_{mn}, \pi_{mn}) \end{pmatrix}$$

Step -2: Determination of Weights Criteria

The weighted criteria is

$$w_j = \frac{S_j}{\sum_{j=1}^{n} \overline{S_j}} \; where \; \overline{S_j} = \frac{1}{p} \sum_{j=1}^{n} \Delta_j$$

with the condition that $0 \leq \overline{w_j} \leq 1$ provided that $\sum_{j=1}^{n} \overline{w_j}' \, S = 1$.

Step-3: Utilize the score function to determine uncertainty positive ideal solution and negative ideal solution.

$$C^+ = \left\{ \langle G_j \, , \alpha_i{}^+, \pi_i{}^+ \rangle \right\} where \; (\alpha_i{}^+, \pi_i{}^+) = (1,0), i \in F_1$$

$$C^- = \left\{ \langle G_j \, , \alpha_i{}^-, \pi_i{}^- \rangle \right\} where \; (\alpha_i{}^-, \pi_i{}^-) = (1,0), i \in F_2$$

and using Monte-Carlo fuzzy aggregate fuzzy value.

Step 4 : Use the distance formula to calculate distance between two ideal solutions

$$M^+(C_i) = \left[\frac{1}{2} \sum_{i=1}^{n} w_j |1 - \alpha_i| + \left| \pi_i{}^{\frac{1}{2}} \right| \right]^{\frac{1}{2}}$$

$$M^-(C_i) = \left[\frac{1}{2} \sum_{i=1}^{n} w_j |\alpha_i| + \left| 1 - \pi_i{}^{\frac{1}{2}} \right| \right]^{\frac{1}{2}}$$

where $i = 1,2, \dots, m$ and $j = 1,2, \dots, n$.

Step 5: Calculation of relative closeness

The relative closeness degree is calculated as

$$M(C_i) = \frac{M^-(C_i)}{M^+(C_i) + M^-(C_i)}$$

The alternative with maximum relative closeness degree is taken as the best alternative among all other alternative.

Problem Statement: Consider, the people want to choose the best hospital in 4 various districts and 4 familiar hospitals for the best treatment of brain tumor with respect to age specifications.

D_1 = Patients from hospitals in A zone

D_2 = Patients from hospitals in B zone

D_3 = Patients from hospitals in C zone

D_4 = Patients from hospitals in D zone

To choose the first one, we need to consult some expect doctors and their opinions are represented by the following four criterions.

I_1 : Highly in fracture and well-equipped instruments

I_2 : Highly qualified doctors

I_3 : Patient Approach
I_5 : Treatment Methodolgy

For the selection of best alternative, we apply our proposed monte carlo entropy measure equation (3) to equation (5)

Step – 1: First we construct the monte carlo fuzzy decision matrix

D_1 =

Age Interval	Clinical Datasets			
	I	II	III	IV
0-10	0.25	0.50	0.75	0.65
10-20	0.10	0.22	0.35	0.45
20-30	0.17	0.25	0.5	0.75
30-40	0.40	0.30	0.60	0.70
40-50	0.60	0.30	0.50	0.55
50-60	0.40	0.30	0.250	0.530
60-70	0.50	0.62	0.750	0.250
70-80	0.45	0.50	0.65	0.750

D_2 =

Age Interval	Clinical Datasets			
	I	II	III	IV
0-10	0.30	0.45	0.50	0.82
10-20	0.40	0.62	0.55	0.92
20-30	0.50	0.64	0.60	0.94
30-40	0.70	0.65	0.75	0.42
40-50	0.80	0.65	0.80	0.52
50-60	0.90	0.72	0.84	0.72
60-70	0.85	0.82	0.35	0.66
70-80	0.10	0.20	0.54	0.64

D_3 =

Age Interval	Clinical Datasets			
	I	II	III	IV
0-10	0.35	0.60	0.70	0.62
10-20	0.40	0.24	0.80	0.84

20-30	0.62	0.30	0.40	0.75
30-40	0.51	0.25	0.60	0.65
40-50	0.21	0.62	0.50	0.45
50-60	0.25	0.40	0.40	0.50
60-70	0.45	0.50	0.30	0.55
70-80	0.50	0.75	0.80	0.65

D_4 =

Age Interval	Clinical Datasets			
	I	II	III	IV
0-10	0.60	0.70	0.65	0.45
10-20	0.50	0.65	0.80	0.85
20-30	0.25	0.30	0.35	0.40
30-40	0.45	0.50	0.02	0.72
40-50	0.80	0.85	0.90	0.88
50-60	0.72	0.84	0.94	0.64
60-70	0.90	0.85	0.40	0.55
70-80	0.40	0.65	0.80	0.45

Step -2: We calculate the weights of each criterian using the four proposed monte carlo entropy measure from equation (7) to equation (9)

Entropy	$\overline{w_1}$	$\overline{w_2}$	$\overline{w_3}$	$\overline{w_4}$
ld_1	0.4098	0.4593	0.1306	0.2356
ld_2	0.4799	0.4406	0.0792	0.2756
$l_{min/max}$	0.7147	0.1843	0.1027	0.2372

Entropy values and weight of criteria

Step 3: By choosing $w_i = 2$ and $k = 2$

D_1	1.618	1.926	1.770	1.662	1.674	1.818	1.627	1.548
D_2	1.643	1.512	1.448	1.493	1.410	1.268	1.449	1.826
D_3	1.574	1.587	1.642	1.658	1.729	1.796	1.720	1.433
D_4	1.572	1.621	1.520	1.548	1.674	1.584	1.620	1.592

Step 4: We calculate the distance alternatives of fuzzy positive and negative ideal solution

ld_1	$M^+(C_i)$ Ascending	$M^-(C_i)$ Highest -Lowest
D_1	1.7053	0.3780
D_2	1.5061	0.5580
D_3	1.6423	0.3630
D_4	1.5913	0.1540

Distance of alternative ld_1

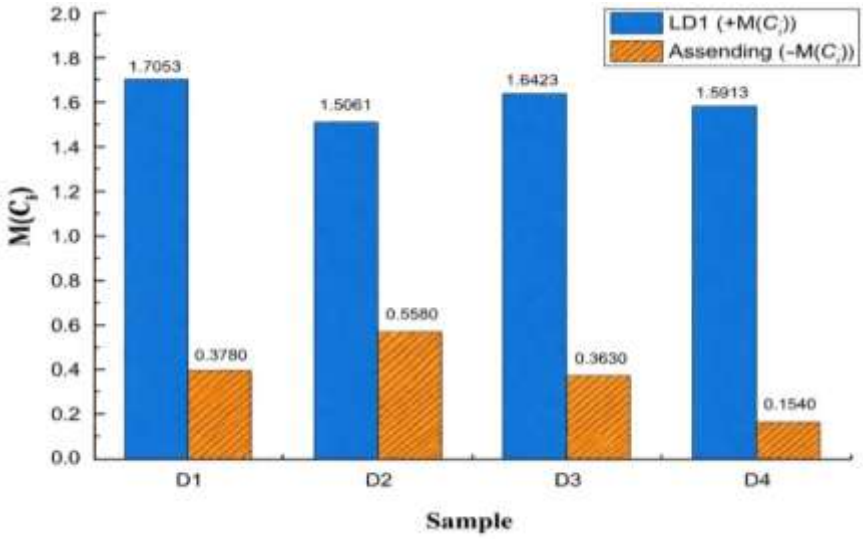

Figure: Positive and Negative for Ideal solutions

ld_2	$M^+(C_i)$ Ascending	$M^-(C_i)$ Highest -Lowest
D_1	1.3804	0.3059
D_2	1.1809	0.4375
D_3	1.2877	0.2846
D_4	1.2477	0.1207

Distance of alternative ld_2

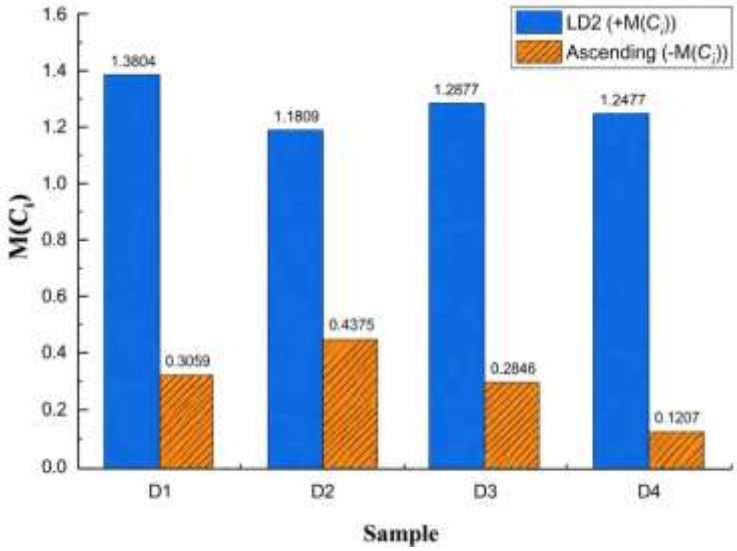

Figure: Positive and Negative for Ideal solutions

$l_{min/max}$	$M^{+}(C_i)$	$M^{-}(C_i)$
D_1	1.3764	0.3051
D_2	1.2156	0.4503
D_3	1.3256	0.2930
D_4	1.2844	0.1243

Distance alternative $l_{min/max}$

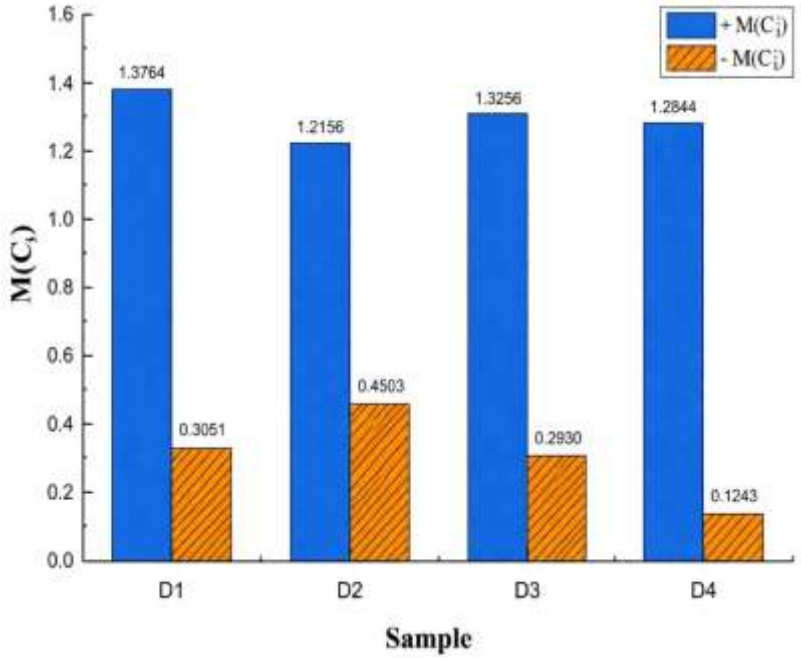

Figure: Distance Alternative for Ideal Solution

Step 5 The degree of relative closeness is calculated as

ld_1	$R(\widetilde{C}_i)$

D_1	0.8185
D_2	0.7296
D_3	0.8189
D_4	0.9117

Distance alternative $l_{min/max}$

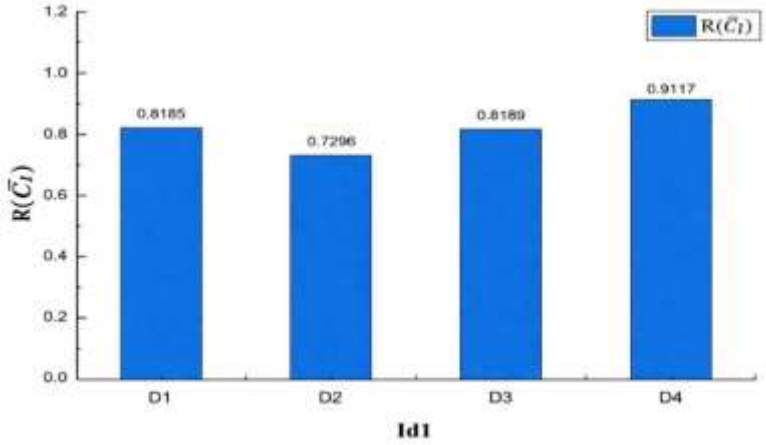

Figure: Relative closeness for Ideal Solution ID_1

ld_2	$R(\widetilde{C}_i)$
D_1	0.8185
D_2	0.7296
D_3	0.8189
D_4	0.9117

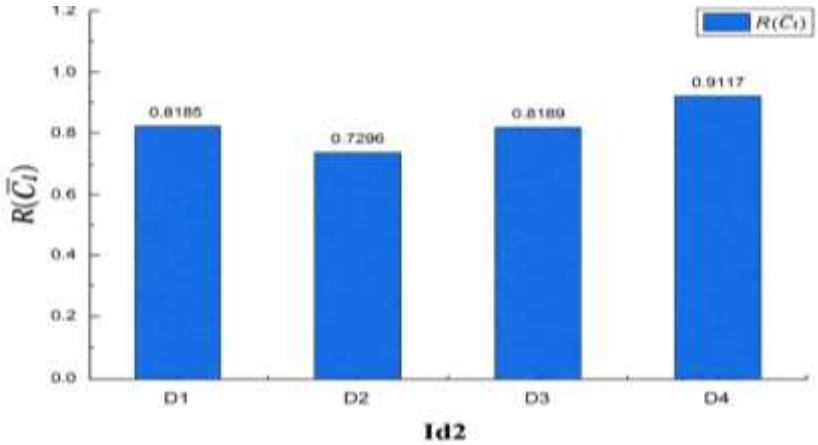

Figure: Relative closeness for Ideal Solution ID_2

$l_{min/max}$	$R(\widetilde{C}_i)$
D_1	0.8185
D_2	0.7296
D_3	0.8189
D_4	0.9117

Degree Of Relative Closeness

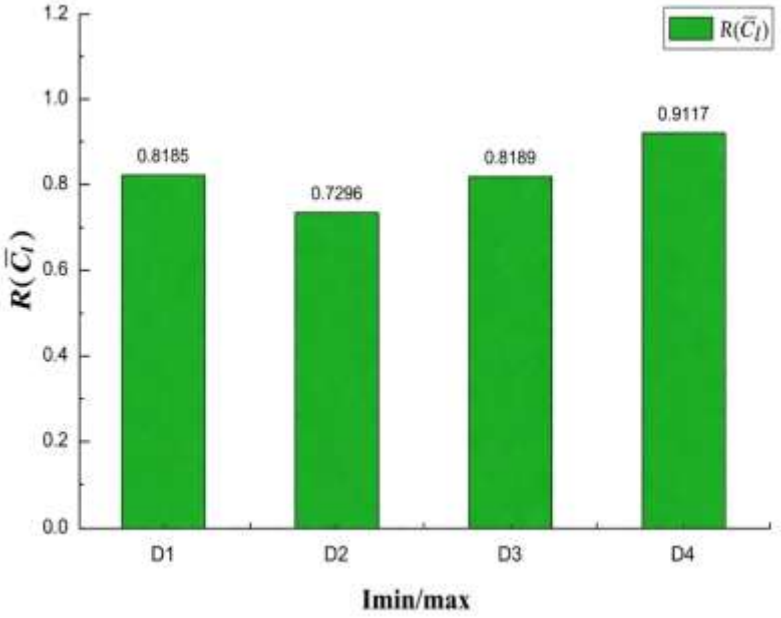

Figure: Relative closeness for Ideal Solution ID_3

Step 6 The Ranking alternative is calculated by

Entropy	Rank	Best alternative
ld_1	$D_4 > D_3 > D_1 > D_2$	$\overline{D_4}$
ld_2	$D_4 > D_3 > D_1 > D_2$	$\overline{D_4}$
$l_{min/max}$	$D_4 > D_3 > D_1 > D_2$	$\overline{D_4}$

Hence D_4 is the best alternative for the best treatment of brain tumor and the maximum value of D_4 in step 3 is 1.674 whose corresponding age interval is 40-50.

Conclusion

The current study introduced a new decision support system which combines fuzzy sets, entropy-based weights and Monte Carlo simulation for evaluating and optimizing brain tumor zones under uncertainty conditions. With the use of the new methodology, it was possible to successfully cope with uncertainties and variances which typically accompany such types of clinical and imaging data. As for Monte Carlo simulation, it ensured the reliability of the performed computations by taking stochastic changes into account. As for entropy measures (ld_1), (ld_2) (min/max), they were objective indicators used to establish the significance of each criterion utilized during the evaluation process. In this respect, the rankings computed through the application of all three measures proved equal, yielding an identical result represented by the sequence (D_4 > D_3 > D_1 > D_2). It means that the proposed framework is stable, consistent and robust enough to be employed for clinical purposes. As follows from the rankings, zone (D_4) is the optimal choice as it demonstrates a higher priority among all alternatives considered.

As for the main conclusion to be drawn, it should be emphasized that the combination of fuzzy logic with probabilistic optimization via Monte Carlo simulations represents a powerful technique that allows coping with uncertainty in the field of neuro-oncology. At the same time, the proposed methodology proves more reliable, interpretable and robust than other deterministic approaches. On the clinical level, it will facilitate the identification of critical tumor zones, which will eventually improve diagnostic outcomes and treatment procedures.

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