A STRUCTURAL STUDY OF NRWG*- CLOSED SETS VIA HOMEOMORPHISMS AND IRRESOLUTE MAPPINGS IN NANO TOPOLOGY

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Abstract

In this paper, we introduce a new class of homeomorphisms called Nrwg*-homeomorphisms in nano topological spaces using nano regular weakly generalized closed sets. We define and study Nrwg*-open and Nrwg*-closed functions, which generalize nano open and closed mappings. Various characterizations and fundamental properties of these functions are established, along with their relationships to existing types of mappings. Examples are provided to illustrate the results and the proper inclusions between these classes.

Keywords: Nrwg*-open functions, Nrwg*-closed functions, Nrwg*-homeomorphism, Nrwg*-irresolute functions.

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I- Introduction

The study of topology provides a fundamental framework for understanding the structure of mathematical spaces and the behavior of functions defined on them. In recent years, **nano topology**, developed using the concepts of lower and upper approximations from rough set theory, has emerged as an important generalization of classical topology. This framework is particularly useful in dealing with imprecise and uncertain information, making it applicable in areas such as data analysis, decision-making, and information systems.Within nano topology, various generalized forms of closed sets and continuous functions have been introduced to extend the classical notions and to better capture the structural properties of nano topological spaces. Among these, nano regular weakly generalized closed sets play a significant role in defining broader classes of mappings that relax the strict conditions of continuity and homeomorphism.Motivated by these developments, this paper introduces a new class of homeomorphisms, namely **Nrwg*-homeomorphisms**, defined using nano regular weakly generalized closed sets. These mappings provide a natural generalization of existing nano homeomorphisms and offer a more flexible approach to studying equivalence between nano topological spaces. In addition, we define and investigate **Nrwg*-open** and **Nrwg*-closed functions**, which extend the concepts of nano open and nano closed mappings.Furthermore, we introduce a new type of generalized continuity, called **contra Nrwg*-continuous functions**, which is based on the behavior of inverse images of nano regular weakly generalized closed sets. This concept broadens the scope of contra continuity in nano topological settings and helps in analyzing the interplay between different classes of functions.The main objective of this paper is to explore the fundamental properties and characterizations of these newly defined functions and to examine their relationships with existing types of mappings in nano topology. Several results are established to demonstrate inclusion relationships among these classes, and suitable examples are provided to justify the properness of these inclusions.This work contributes to the ongoing development of nano topology by introducing new structures and opening avenues for further research in both theoretical and applied aspects of generalized topological spaces.

II. PRELIMINARIES

Definition: 2.1

R is an equivalence relation on U known as the Indiscernibility relation. Let U be a non-empty finite set of things called the universe. After that, U is separated into classes with disjoint equivalencies. It is argued that elements that are indistinguishable from one another belong to the same equivalency class.

finition: 2.2

"Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U names as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space**.**

Let $X \subseteq U$. Then

- The lower approximation of X with respect to R is the set of all the objects which can be for certainly classified as X with respect to R is denoted by $L_R(X)$.

$L_R(X)=\cup_{x\in U}\{R(X):R(X)\subseteq X\}$

- The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$.

$U_R(X)=\cup_{x\in U}\{R(X):R(X)\cap X\neq\phi\}$

- The boundary region of X with respect to R is the set of all objects which can be classified either as X nor as not-X with respect to R and is denoted by $B_R(X)$.

$B_R(X)=U_R(X)–L_R(X)$"

Definition:2.3

"Let U be the universe ,R be an equivalence relation on U and$\boldsymbol{\tau}_R(X)=\{U,\boldsymbol{\phi},L_R(X),U_R(X),B_R(X)\}$ where $X\subseteq U$. Then $\boldsymbol{\tau}_R(X)$ satisfies the following axioms:

- U and $\boldsymbol{\phi}\in\boldsymbol{\tau}_R(X)$.
- The union of the elements of any sub collection of $\boldsymbol{\tau}_R(X)$ is in $\boldsymbol{\tau}_R(X)$.
- The intersection of the elements of any finite sub collection of$\boldsymbol{\tau}_R(X)$ is in $\boldsymbol{\tau}_R(X)$. Then $\boldsymbol{\tau}_R(X)$ is called the Nano topology on U with respect to X,$(U,_R(X))$ is called the Nano topological space."

Definition: 2.4

"If $(U, \boldsymbol{\tau}_R(X))$ is a Nano topological space with respect to X where $X\subseteq U$ and if $A\subseteq U$, then, Nano interior of a set A is defined as the union of all the Nano open sets contained A and it is denoted by N-int(A)."

Definition: 2.5

"If (U, τ_R(X)) is a Nano topological space with respect to X where X⊆U and if A⊆U, then Nano closure of a set A is defined as the intersection of all nano closed sets containing A and it is denoted by N-cl(A)."

Definition: 2.6
Consider (U,′(Y)) and (W,τ_R''(Z)) be any two nano topological spaces. If the inverse image of each nano open set in W is nano open in U, then the mapping h: (U,τ_R'(Y)) □(W,τ_R''(Z)) is stated as nano continuous.

Definition: 2.7
Consider a function h from V to W.

- Each nano open set in W has an inverse image in V that is nano pre-open ⟹ g is nano pre-continuous.
- Each nano open set in W has an inverse image in V that is nano semi-open⟹ gisnano semi continuous.
- Each nano open set in W has an inverse image in V that is nano regular open ⟹g is nano regular continuous.
- Each nano open set in W has an inverse image in V that is nano □-open⟹ g is N□(pre semi)-continuous.
- Each nano open set in W has an inverse image in V that is nano □ -open⟹g is N □ -continuous.
- Each nano open set in W has an inverse image in V that is Nb-open ⟹g is Nb-continuous.
- Each nano open set in W has an inverse image in V that is nano □-open ⟹g is N□-continuous.
- Each nano open set in W has an inverse image in V that is Ng-open ⟹g is Ng-continuous.

Definition: 2.8
Consider a function h fromV to W.

- Every nano semi-closed set in W has an inverse image in V that is nano semi-closed⟹
g is nano irresolute.
- Every Nδg-closed set in W has an inverse image in V that is Nδg-closed ⟹g is Nδg-irresolute.
- Every Nsg- closed set in W has an inverse image in V which is Nsg-closed ⟹g is Nsg-irresolute.
- Every Nrg- closed set in W has an inverse image in V which is Nrg-closed ⟹ g is Nrg-irresolute.

Definition: 2.9
If every nano open set in U has an image that is also nano open in W, then the function h: (U,′(Y)) □(W,τ_R''(Z)) is referred to be a nano open map.

Definition: 2.10
If each nano closed set in U has an image that is also nano closed in W, then the function
h : (U,τ_R'(Y))□□(W,τ_R''(Z)) is referred to be a nano closed map.

III- Nano Regular Weakly Generalized*-Continuous Functions

Definition: 3.1
Consider (V,′(Y)) and (W,τ_R''(Y)) be any two nano topological spaces. If the inverse image of every nano open set in W is Nrwg*-open in V, the mapping h: (V,′(Y)) □(W,τ_R''(Y)) is stated as nano regular weakly generalized*-continuous.

Example: 3.2
Let V={s, v, l, m}, Y ={s,v}, V/R ={{s},{l},{v, m}}, τ_R(Y)= {V, □,{s},{v,m},{s,v,m}}.
Nrwg*- open sets={V, □,{s},{v,m},{s,v,m}}. Let W={12,6,5,7},Z={5,7},W/R'={{6},{7},{12,5}}.
τ_R'(Z)={W,□,{7},{12,5},{12,5,7}}. Nrwg*-open sets={W,□,{7},{12,5},{12,5,7}}. Consider h:V□W as h(s)=7, h(v)=5, h(l)=6, h(m)=12 which implies h^{-1}(W)=V,h^{-1}(□)=(□), h^{-1}({7})={s}, h^{-1}({12,5}) = {v,m}, h^{-1}({12,5,7})={m,v,s}.(i.e)The inverse image of every nano open set in W is Nrwg* -open in V. Hence h is Nrwg*-continuous.

Theorem: 3.3
Every nano regular continuous function is Nrwg*-continuous but not conversely.

Proof:
Given that a mapping h from V to W be nano regular continuous.Then the inverse image of every nano open set in W is nano regular open in V. "Every nano regular open set is Nrwg*-open". Therefore, the inverse image of every nano open set in (W,″(Z))is Nrb-open in (V,τ_R'(Y)) Which implies Nrwg*-continuous.

Example: 3.4
Consider V={y, v, a, k}, Y ={y, v}, V/R={{y},{a},{v, k}}. τ_R(Y)= {V, □,{y},{v,k},{y,v,k}}.
Nano regular open sets = {V, □,{y},{v, k}}. Nrwg*-open sets={V, □,{y},{v,k},{y,v,k}}. Let W={21,22,23,24}, Z={23,24},W/R'={{22},{24},{21,23}}. τ_R'(Z)={W, □,{24},{21,23},{21,23,24}}.
Nano regular open sets={W, □,{24},{21,23}}. Consider h:V□W as h(y)=24, h(v)=23, h(a)=22, h(k)=21 which implies h^{-1}(W)=V, h^{-1}(□)=(□), h^{-1}({24})={y}, h^{-1}({21,23})={v,k}, h^{-1}({21,23,24})={k,v,y}.From h^{-1}({21,23,24}) = {k,v,y},this is not a nano regular open set in(V,τ_R(Y)). □This mapping is Nrwg*-continuous but not a nano regular continuous.

Theorem: 3.5

Every Nrwg*-continuous function is nano continuous but not conversely.

Proof:

Given that a mapping g from V to W be Nrwg*-continuous, Which implies that the inverse image of every nano open in W is Nrwg*-open inV. “Every Nrwg*-open set is nano open”.The inverse image of every nano open in (V,′(Y)) is nano open in (U,τ_R(X)). Hence f is nano continuous.

Example:3.6

Let U ={5,6,7,8}, X={5,6}, U/R = {{5},{6},{7,8}}. τ_R(X)={U,∅,{5,6}}, Nrb-open sets = {U,∅}.Let V={10,20,30,40},Y={30,40},V/R'={{20},{40},{10,30}}. $\tau_{R'}$(Y)={V,□,{40},{10,30}, {10,30,40}}, Consider h:U □V as h(50)=10,h(60)=30,h(7)=20,h(8)=40,thenh^{-1}(V)=U,h^{-1}(□)=(□), h^{-1}({40})={8},h^{-1}({10,30})={5,6},h^{-1}({10,30,40})={5,6,8}. Here h^{-1}({40}) = {8}, h^{-1}({10,30}) ={5,6},h^{-1}({10,30,40}})={5,6,8}which are not Nrwg*-open in (U,τ_R(X)). This mapping is nano continuous but not Nrwg*-continuous.

Theorem: 3.7

Every Nrwg*-continuous function is Ng- continuous but not conversely.

Proof:

Given that h from(V,$\tau_{R'}$(Y)) to(W,$\tau_{R''}$(Z)) be Nrwg*-continuous, Which implies that the inverse image of every nano open set in W is Nrwg*-open in V. “Every Nrwg*-open set is Ng-open”. The inverse image of every nano open set in (V,′(Y)) is Ng-open in (U,τ_R(X)). Hence f is Ng-continuous.

Example:3.8

Let V={k, j, y, f}, Y={k, j}, V/R ={{k},{y},{j, f}}.τ_R(Y)= {V, □,{k},{j,f},{k,j,f}}.Nrwg*- open sets={V,□,{k},{j,f},{k,j,f}}.Ng-open sets={V, □,{k},{j},{f},{k,j},{k,f},{j,f},{k,j,f}}. Let W={2,4,6,8},Z={6,8},W/R'={{4},{8},{2,6}}. $\tau_{R'}$(Z)={W,□,{8},{2,6},{2,6,8}}. Consider h:V □W as h(k)=2,h(j)=8,h(y)=4,h(f)=6, then h^{-1}(V)=U,h^{-1}(□)=(□), h^{-1}({8})={j}, h^{-1}({2,6}) ={k,f},h^{-1}({2,6,8})={k,f,j}. Hereh^{-1}({8})={j},h^{-1}({2,6})={k,f} which are not Nrwg*-open in(V,τ_R(Y)).This mapping is Ng-continuous but not Nrwg*-continuous.

Theorem: 3.9

Every Nano alpha-continuous function is Nrwg*-continuous but not conversely.

Proof:

Given that h:(V,$\tau_{R'}$(Y)) □(W,$\tau_{R''}$(Z)) be Nano alpha-continuous, Which implies that the inverse image of every nano alpha open set in W is Nrwg*-open in V. “Every Nano alpha-open set is Nrwg*-open. Hence f is Nrwg*-continuous.

Example: 3.10

Let V={2, 8,16, 24}, Y={2, 8}, V/R={{2},{16},{8, 24}}.τ_R(Y)= {V, □,{2},{8,24},{2,8,24}}. Nrwg*-open sets={V, □,{2},{8,24},{2,8, 24}}. Nano alpha open sets={V, □,{2},{8},{ 24},{2,8}, {2,16}, {2,24},{8,24},{2,8,16},{2,8,24}, {2,16,24},{8,16,24}}. Let W={3,6,9,12}, Z={9,12},W/R'= {{6},{12},{3,9}}, $\tau_{R'}$(Z)={W,□,{12},{3,9},{3,9,12}}. Consider h:V□W as h(2)=3, h(8)=12, h(16)= 9, h(24)=6, then h^{-1}(V)=U, h^{-1}(□)=(□),h^{-1}({12})={8},h^{-1}({3,9})={2,16},h^{-1}({3,9,12})={2,16,8}. Sinceh^{-1}({12})={8},h^{-1}({3,9})={2,16},h^{-1}({3,9,12})={2,16,8}which are not Nrwg*-open inV. □This mapping is Nrwg*- continuous but not Nano alpha-continuous.

Theorem: 3.11

Every Ng-continuous function is Nrwg*-continuous but not conversely.

Proof:

Given that a mapping from V to W be Ng-continuous.Which implies that the inverse image of every nano g open set in W is Nrwg*-open in V. “Every Ng-open set is Nrwg*-open”. Therefore, the inverse image of every nano open set in W is Nrwg*-open in V.h is Nrwg*-continuous.

Example: 3.12

Let V={y,s, k, d},Y ={y,s}, V/R={{y},{k},{s, d}}. τ_R(Y)= {V, □,{y},{s,d},{y,s,d}}. Ng- open sets={V,□,{y},{s,d},{y,s,d}}. Nrwg*-open sets={V,□,{y},{s},{d},{y,s},{y,k},{y,d},{s,d}, {y,s,k},{y,s,d},{y,k,d},{s,k,d}}. Let W = {2,5,7,9}, Z = {7,9}, W/R'= {{2},{9},{5,7}},$\tau_{R'}$(Z)= {W, □, {9},{2,7},{2,7,9}}. Define h:V□W as h(y)=2,h(s)=9, h(k)=5,h(d)=7, then h^{-1}(V)=U, h^{-1}(□)=(□), h^{-1}({9})={s},h^{-1}({2,7})={y,d},h^{-1}({2,7,9})={y,d,s}. Here h^{-1}({9}) ={s}, h^{-1}({2,7}) ={y,d} which are not Nrwg* open in (V,τ_R(Y)).This mapping is Nrwg*-continuous but not Ng-continuous.

Theorem: 3.13

Every Nrg-continuous function is Nrwg*-continuous but not conversely.

Proof:

Consider a mapping h from V to W be Nrg-continuous.Which implies that the inverse image of every nano rg open set in W is Nrwg*-openinV. “Every Nrg-open set is Nrwg*-open.”Therefore, the inverse image of every nano rg open set in (W,″(Z)) is Nrwg*-open in (V,$\tau_{R'}$(Y)). h is Nrwg*-continuous.

Example: 3.14

Let V={3,6, 7, 9},Y={3,6}, V/R={{3},{7},{6, 9}}.τ_R(Y)={V, □,{3},{6,9},{3,6,9}}.Nrg- open sets={V,□,{3},{6,9},{3,6,9}}. Nrwg*-open sets={V, □,{3},{6},{9},{3,6},{3,9}, 6,9},{3,6,7},{3,6,9}, {3,7,9}}. Let W = {11,12,13,14}, Z = {13,14}, W/R'= {{12},{14},{11,13}}.τ_R'(Z)={W, □,{14}, {11,13}, {11,13,14}}. Consider h:V □W as h(3)=11,h(6)= 14,h(7)=12,h(9)=13, then h^{-1}(W)=V, h^{-1}(□)=(□),h^{-1}({14})={6},h^{-1}({11,13})={3,9},h^{-1}({11,13,14})={3,9,6}. Since h^{-1}({14})= {6},h^{-1}({11,13})={3,9}which are not Nrg-open in (V,τ_R(Y)). This mapping is Nrwg*-continuous but not Nrg-continuous.

Theorem: 3.15

A mapping h from V to W is Nrwg*-continuous if and only if the inverse image of every nano closed set in W is Nrwg*-closed in V.

Proof:

Necessary part:

Given that h: (V,′(Y))□ (W,τ_R''(Z)) is Nrwg*-continuous function. Consider F be nano closed in (W, τ_R''(Z)). (i.e)W-Fis nano open in (W,″(Z)). ⟹h^{-1}(W-F)is Nrwg*-open in (V,τ_R'(Y)),since h is Nrwg*-continuous. (i.e) h^{-1}(W - F) = h^{-1}(W) – h^{-1}(F) = V – h^{-1}(F) is Nrwg*-open in (V,τ_R'(Y)). Therefore, h^{-1}(F) is Nrwg*-closed in (V,τ_R'(Y)).

Sufficient part:

Suppose that each nano closed set in (W,″(Z)) has an inverse image that is Nrwg*-closed in V. Let G be nano open in (W,″(Z)), then W-G is nano closed in (W,τ_R''(Z))□h^{-1}(W-G)is Nrwg*-closed in(V,τ_R'(Y)). (i.e)h^{-1}(W-G)=h^{-1}(W)–h^{-1}(G)=V–h^{-1}(G) is Nrwg*-closed in (V,τ_R'(Y)).h^{-1}(G) is Nrwg*-open in (V,τ_R'(Y)). Hence the inverse image of every nano open set G in (W,″(Z)) is Nrwg*-open in (V,τ_R'(Y)).(i.e) h is Nrwg*-continuous on (V,τ_R'(Y)).

Theorem: 3.16

Consider a mapping h from V to W is a Nrwg*-continuous then h(Nrwg*cl(S)) ⊆ Ncl((h(S)) for every S ⊆ V.

Proof:

Given that h is Nrwg*- continuous and S be the subset of V. Therefore, h(S) ⊆ W.h^{-1}(Ncl (h(S))) is Nrwg* closed in V, because h is Nrwg*-continuous and Ncl(h(S)) is nano closed in W. Since h(S)⊆Ncl(h(S)), S⊆h^{-1}(Ncl(h(S))). Here h^{-1}(Ncl(h(S))) is a Nrwg*-closed set containing S. But Nrwg*cl(S) is the smallest Nrwg*closed set containing S. □Nrbcl(S)⊆h^{-1}(Ncl(h(S))).
(i.e) h(Nrwg*cl(S)) ⊆Ncl(h(S)).

IV- Nano Regular Weakly Generalized*-Irresolute Functions

Definition:4.1

A mapping h from V to W is Nano regular weakly generalized*- irresolute (Nrwg*-irresolute) function on V if every nano regular weakly genealized-closed set in W has an inverse image in V which is nano regular weakly generalized*-closed.

Example: 4.2

ConsiderV={7,5,11,6},Y={7,5,11}, V/R ={7},{5},{11,6}}.τ_R(Y)={V,∅,{7,5},{11,6}}. Nrwg* -closed sets={V,∅, {7,5},{11,6}}.Let W= {p,b,j,d},Z={p,b,j},W/R'={p},{j},{b,d}}.τ_R'(Z) ={W,∅,{p,j},{b,d}}. Nrb- closed sets={W,∅,{p,j},{b, d}}. Consider h:V□W as h(7)=p, h(5)=j,h(11)=b,h(6)=d, thenh^{-1}(W)=V, h^{-1}(□)=(□),h^{-1}({p,j})={7,5},h^{-1}({b,d})={11,6}. (i.e) The inverse image of every Nrwg*-closed in W is Nrwg*-closed in V. Hence h is Nrwg*-irresolute in V.

Theorem: 4.3

A function h from V to W is Nrwg*-irresolute if and only if the inverse image of every Nrwg*-open set in (W,″(Z)) is Nrwg*-open in (V,τ_R'(Y)).

Proof:

Necessary part:

Given that h from V to W is Nrwg*-irresolute and H is Nrwg*-open in (W,″(Z)). Then W – H is Nrwg*-closed in (W,″(Z)). Since H is Nrwg*-irresolute, the inverse image of every Nrwg*-closed in W is Nrwg*-closed inV. ⇒h^{-1}(W–H) is Nrwg*-closed in (V,τ_R'(Y)). (i.e)h^{-1}(W–H)=h^{-1} (W)–h^{-1}(H)=V–h^{-1}(H)(∵h^{-1}(W)=V). ⇒V–h^{-1}(H) is Nrwg*-closed in (V,τ_R'(Y)). ⇒h^{-1}(H) is Nrwg*-open in V. ∴The inverse image of every Nrwg*-open set in W is Nrwg*-open in V.

Sufficient part:

Assume that every Nrwg*-open set in(W,τ_R''(Z)) has an inverse image that is also Nrwg*-open in (V,τ_R'(Y)). Suppose H be Nrwg*-closed in W. Then W–H is Nrwg*-closed in W.
⇒h^{-1}(W–H) is Nrwg*-open in V.(i.e)h^{-1}(W–H)=h^{-1}(W)–h^{-1}(H)=V–h^{-1}(H)(∵h^{-1}(W)=V). ⇒V–h^{-1}(H) is Nrwg*-open in V. ⇒h^{-1}(H) is Nrwg*-closed in V.⇒ Each Nrwg*-closed set in (W,τ_R''(Z)) has an inverse image which is also Nrb-closed in (V,τ_R'(Y)). Therefore, h is Nrwg*-irresolute in (V,′(Y)).

Theorem: 4.4

Consider a mapping g from U to V be a Nrwg*-irresolute function and h from V to W be a Nrwg*-irresolute function. Then the composition is Nrwg*-irresolute function.

Proof:

Consider E be a Nrwg*-closed in (W,″(Z)). Given that h is a Nrwg*-irresolute function from V→W implies h^{-1}(E) is Nrwg*-closed in V. Given that g is a Nrwg*-irresolute function from U→ V implies $g^{-1}(h^{-1}$(E)) is Nrwg*-closed in (U,τ_R(X)). ⇒ $(h \circ g)^{-1}$(E) is Nrwg*-closed in (U,τ_R(X)). Hence $h \circ f$ is Nrwg*-irresolute function.

Theorem: 4.5

Consider a mapping g from U to V be a Nrwg*-irresolute function and let h from V to W be a Nrwg*-continuous function. Then the composition is Nrwg*-continuous function.

Proof:

Given that E is nano closed in (W,τ_R''(Z)). Which implies h^{-1}(E) is Nrwg*-closed in (V,τ_R'(Y)), because h is a Nrwg*-continuous function from V → W. ⇒$g^{-1}(h^{-1}$(E)) is Nrwg*-closed in U, while g is a Nrwg*-irresolute function from U→V.⇒ $(h \circ g)^{-1}$(E) is Nrwg*-closed in (U,τ_R(X)). Hence $h \circ g$ is Nrwg*- continuous function.

Definition: 4.6

If the image of every nano open set in V is Nrwg*-open in W, a function h from V to Wis referred to be Nrwg*-open map.

Example: 4.7

Consider V={d, q, v, k}, Y={d, q}, V/R ={d, v,k},{q}. τ_R(Y) ={V,□,{q},{d,v,k}}.Let W={t,r,s,e},Z={t,r,s},W/R'={t,r,e},{s}. τ_R'(Z)={W,□,{s},{t,r,e}}, Nrwg*-open sets={W, □,{s}, {t,r,e}}. Suppose that g:V□W as g(d)=t,g(q)=s,g(v)=r,g(k)=e,g(U)=V,g(□)= □, then g({q})={s}, g({d,v,k}) ={t,r, e},g(V) =W,g(□)=□. □Every nano open set in V has an image that is Nrwg*-open in W. Hence g is Nrwg* -open.

Definition: 4.8

If the image of every nano closed set in V is Nrwg*-closed in W, a function h from V to W is referred to be Nrwg*-closed map.

Example: 4.9

Consider V={7,8,9}, Y ={7,8}, V/R ={7,9},{8}. τ_R(Y)={V, □,{8},{7,9}}, Nano closed sets ={V, □, {8},{7,9}}. Let W={l,o,x},Z={o,x},W/R'={l,o},{x}. τ_R'(Z)={W,□,{x},{l,o}}, Nrwg* -closed sets={W, □,{x},{l,o}}. Suppose that g: V□W as g(7) =l, g(8) = x, g(9) = o, g(U) = V, g(□) = □, then g({8}) = {x}, g({7,9}) = {l, o}. □Each nano closed set in V has an image that is Nrwg*-closed in W.
Hence g is Nrwg*-closed.

Theorem: 4.10

Suppose that the two mappings are g from U to V and h from V to W such that their composition hog: (U,τ_R(X))□(W,τ_R''(Z)) is a Nrwg*-closed mapping. Then

- If g is nano continuous and surjective ⇒ h is Nrwg*-closed map.
- If h is Nrwg*-irresolute and injective ⇒g is Nrwg*-closed map.

Proof:

(i) Consider A is a nano closed set in V. ⇒g^{-1}(A) is nano closed in U, while g is nano continuous. Given that hog:(U,τ_R(X))□(W,τ_R''(Z)) is Nrwg*-closed implies $(h\ og)(g^{-1}$(A)) is Nrwg*-closed in (W,τ_R''(Z)).(ie) h(A) is Nrwg*-closed in (W,τ_R''(Z)) and also g is surjective. Hence the image of a nano closed set A in (V,′(Y)) is Nrwg*-closed in (W,τ_R''(Z)). □h:(V,τ_R'(Y)) □(W,τ_R''(Z)) is Nrwg*-closed.

(ii) Consider B be any nano closed set in U. From $h\ o\ g$:(U,τ_R(X))□(W,τ_R''(Z)) is Nrwg*-closed implies $(h\ o\ g)$(B) is Nrwg*-closed in (W,τ_R''(Z)). Since h is Nrwg*-irresolute implies $h^{-1}(hog)$(B) is Nrwg*-closed in (V,τ_R'(Y)). (ie) g(B) is Nrwg*-closed in (V,τ_R'(Y)), Since the function h:(V,τ_R'(Y)) □
(W,″(Z)) is injective.Therefore, Nano closed set B in U has an image which is Nrwg*-closed in V. Hence g is Nrwg*-closed map.

Theorem: 4.11

If a function h from V to W is Nrwg*-closed function, then Nrwg*cl(h(A)) ⊆ h(Ncl(A)) for every subset A of (V,τ_R'(Y)).

Proof:

Given h:(V,τ_R'(Y))□(W,τ_R''(Z)) be a Nrwg*-closed function and A⊆V.Which implies that Ncl(A) is nano closed in (V,τ_R'(Y)) and f(Ncl(A)) is a Nrwg*-closed set in (W,τ_R''(Z)). Now A ⊆ Ncl(A)⇒h(A) ⊆h(Ncl(A)) ⇒h(Ncl(A)) is a Nrwg*-closed set containing h(A).But Nrwg*cl(h(A)) is the smallest Nrwg*-closed set containing h(A).⇒Nrwg*cl(h(A))⊆h(Ncl(A))

Theorem: 4.12

If a mapping g from U to V is Nrwg*-closed ,h from V to W is Nrwg*-closed map and V is T_{Nrb} space then its composition h:(U,τ_R(X)) □(W,τ_R''(Z)) is Nrwg*-closed map.

Proof:
Consider A be a nano closed set in (U,(X)). Given g is Nrwg*-closed, then g(A) is Nrwg*-closed in V. Given that V is T_{Nrb} space which implies g(A) is Nano closed in (V,τ_R'(Y)). Since h is Nrwg*-closed, then h(g(A)) is Nrwg*-closed in (W,τ_R''(Z)). □$h\ o\ g$: (U,(X)) □(W,τ_R''(Z)) is Nrwg*-closed map. Hence the composition of Nrwg*-closed maps is Nrwg*-closed map.

Theorem: 4.13
A function h from V to W is Nrwg*-open function, then g(Nint(A)⊆Nrwg*int(g(A)) for each subset A of (V,τ_R'(Y)).

Proof:
Assume that h: (V,′(Y))□(W,τ_R''(Z)) be the Nrwg*-open function. Let A ⊆ V. Then Nint(A)⊆A. Then Nint(A) is nano open in(V,τ_R'(Y)).Since h: (V,τ_R'(Y))□(W,τ_R''(Z)) is Nrwg*-open implies h(Nint(A))is Nrwg*-open in (W,τ_R''(Z)). From Nint(A)⊆A. Which implies h(Nint(A)⊆A. ⇒h(Nint(A)) is the Nrwg*-open set contained in h(A).Since Nrwg* int(h(A)) is the largest Nrwg*-open set contained in h(A). Hence h(Nint(A)) ⊆ Nrwg*int(h(A)).

Theorem: 4.14
If a mapping g from U to V is Nrwg*-open map, h from V to W is Nrwg*-open map and V is T_{Nrb} space then its composition hog from U to W is Nrwg*-open map.

Proof:
Let A be a nano open set in(U,τ_R(X)). Since g is Nrwg*-open, then g(A) is Nrwg*-open in (V,τ_R'(Y)). Given that V is T_{Nrb} space which implies g(A) is nano open set in (V,τ_R'(Y)). Since h is Nrwg*-open, then h(g(A)) is Nrwg*-closed in (W,τ_R''(Z)).□$h\ o\ g$: (U,τ_R(X)) □(W,τ_R''(Z)) is Nrwg*-open map. Hence the composition of Nrwg*-open maps is Nrwg*-open map.

V-Conclusion

In this study, we introduced and analyzed the concepts of **nano regular weakly generalized* continuous functions** and **nano regular weakly generalized*-irresolute functions** in nano topological spaces. These functions extend the framework of continuity by incorporating weaker and more generalized conditions based on nano open and closed sets.It is observed that nano regular weakly continuous functions form a broader class than nano continuous functions, allowing more flexibility in handling irregular structures within nano topology. Similarly, nano irresolute functions preserve the structure of nano closed sets under inverse images, providing an important tool for studying function behavior between nano spaces. We established relationships between these functions and other existing types of nano continuous mappings, highlighting inclusion properties and non-equivalence through suitable examples. The results demonstrate that these generalized forms play a significant role in the deeper understanding of nano topological structures. Overall, the introduction of these concepts contributes to the expansion of nano topology and opens pathways for further research, particularly in applications involving data reduction, decision systems, and applied sciences.

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