



## Prime Cordial Labeling of Contact Networks: An Environmental Application to Epidemic Spread in Aquatic Populations

L. N. Narayanan<sup>1,2</sup>, V. Malarvizhi<sup>1,2</sup>, V. Sudhakar<sup>1,2</sup>, P. Vishnu Prakash<sup>3</sup>, N. Avinash<sup>4</sup>, S. Kalaiarasi<sup>4</sup>, V. Balaji<sup>4\*</sup>

### Abstract

A **prime cordial labeling** of a graph  $G$  is a bijection  $f:V(G)\rightarrow\{1,2,\dots,|V(G)|\}$  such that each edge  $uv\in E(G)$  is assigned label 1 if  $\gcd(f(u),f(v))=1$  and 0 if  $\gcd(f(u),f(v))>1$ ; further the number of edges labeled 0 and 1 differ by at most 1. If a graph admits prime cordial labeling, then it is called a prime cordial graph. We prove that the Durer graph, Heawood graph, Frucht graph, Tietze graph, hypohamiltonian graph, cubic graph with 12 vertices, and crown graph are prime cordial graphs. Also we prove that the Herschel graph, Wagner graph, Moser spindle graph, and truncated tetrahedron graph are not prime cordial graphs.

<sup>1</sup>Research Scholar, Department of Mathematics, Sacred Heart College, Tirupattur – 635601, Tamil Nadu, India (Affiliated to Thiruvalluvar University, Serkadu, Vellore – 632115, Tamil Nadu, India)

<sup>2</sup>Department of Mathematics, Jerusalem College of Engineering, Chennai – 600100, Tamil Nadu, India

<sup>3</sup>Department of Mathematics, Arunai Engineering College, Tiruvannamalai – 606603, Tamil Nadu, India

<sup>4</sup>Department of Mathematics, Sacred Heart College (Autonomous), Tirupattur – 635601, Tamil Nadu, India  
Emails: lnnarayanan2020@gmail.com<sup>1</sup>; arumugammalar@gmail.com<sup>2</sup>; avinashprofess@gmail.com<sup>3</sup>; pulibala70@gmail.com<sup>4</sup>

**Corresponding Author\*:** V. Balaji, Department of Mathematics, Sacred Heart College (Autonomous), Tirupattur – 635601, Tamil Nadu, India,

**Keywords:** Prime cordial labeling, Graph labeling, Durer graph, Heawood graph, Frucht graph, Tietze graph, Crown graph

## 1. Introduction

Prime cordial labeling is an important variation of cordial labeling in graph theory, combining ideas from graph labeling and elementary number theory. The concept of cordial labeling was introduced by Cahit [3] as a weaker alternative to graceful and harmonious labeling. Building on this idea, Sundaram, Ponraj, and Somasundaram [14] introduced prime cordial labeling, where vertices are assigned labels so that the induced edge labels, determined by relative primality, satisfy cordial balance conditions.

Since its introduction, several researchers have investigated prime cordial labeling for different families of graphs. Aljouiee [1] studied structural properties and provided results for various graph classes. Babujee and Shobana [2] examined prime and prime cordial labelings for selected special graphs. Kang et al. [7] contributed earlier results on labeling techniques. Sugumaran and Mohan [12], and Sugumaran and Prakash [13], extended results to theta graphs and related families. Parthiban and Sharma [10] provided a survey on prime cordial and divisor cordial labeling. Naeem et al. [9] explored applications in pattern recognition of knight graphs.

Overall, the existing literature demonstrates steady progress in characterizing graph classes that admit prime cordial labeling. Despite these developments, many graph families remain unexplored. These include A-cordial labeling, product cordial labeling, total product cordial labeling, divisor cordial labeling, sum divisor cordial labeling, and prime cordial labeling, among others. First we recall the basic definitions of labeling.

## 2. Preliminaries

**Definition 2.1 (Labeling)** [6]. A labeling of a graph  $G$  is an assignment of integers to the vertices or edges or both.

**Definition 2.2 (Binary Vertex Labeling)** [3]. Let  $G$  be a graph and a labeling map  $h$  assigns each vertex  $w$  of  $G$  as either 0 or 1. Then the graph is called a binary vertex labeling under  $h$ . The induced edge labeling is the absolute difference of labels of end vertices under  $h^*:E(G)\rightarrow\{0,1\}$ .

We denote  $w_h(k)$  and  $e_h(k)$  as the number of vertices and edges of  $G$  having label  $k$  under  $h^*$ , where  $k=0,1$ .

**Definition 2.3 ( $\beta$ -Valuation)** [11]. A  $\beta$ -valuation of a graph  $G$  with  $q$  edges is an injection  $h$  from vertices of  $G$  to  $\{0,1,2,\dots,q\}$  such that each edge  $e=uv$  is assigned the label  $|h(u)-h(v)|$  and the resulting edge labels are distinct.

**Definition 2.4 (Graceful Labeling)** [4]. Let  $h$  be an injection from vertices of  $G$  to  $\{0,1,2,\dots,q\}$ . This is a graceful labeling of  $G$  if we assign each edge  $uv$  the label  $|h(u)-h(v)|$  and the resulting edge labels are distinct.

**Definition 2.5 (Harmonious Labeling)** [5]. The harmonious labeling of a graph  $G$  with  $q$  edges is an injection  $h$  from vertices to  $\{0,1,2,\dots,q-1\}$  such that each edge  $e=uv$  is assigned  $(h(u)+h(v)) \bmod q$  and the resulting edge labels are distinct.

**Definition 2.6 (Cordial Labeling)** [3]. A binary vertex labeling of a given graph  $G$  is a cordial labeling if each edge  $e=uv$  is assigned  $|h(u)-h(v)|$ , the number of vertices having labels 0 and 1 differ by at most 1, and the number of edges having labels 0 and 1 differ by at most 1. If  $G$  admits cordial labeling then it is called a cordial graph.

**Definition 2.7 (Prime Cordial Labeling)** [14]. A prime cordial labeling of a graph  $G$  is a bijection  $f:V(G)\rightarrow\{1,2,\dots,|V(G)|\}$  such that each edge  $uv\in E(G)$  is assigned label 1 if  $\gcd(f(u),f(v))=1$  and 0 if  $\gcd(f(u),f(v))>1$ ; further the number of edges labeled 0 and 1 differ by at most 1. If a graph admits prime cordial labeling, then it is called a prime cordial graph.

## 3. Main Results

We investigate the prime cordial labeling concept for the Durer graph, Heawood graph, Frucht graph, Tietze graph, hypohamiltonian graph, cubic graph with 12 vertices, and crown graph. Further we show that some graphs do not admit prime cordial labeling.

**Theorem 3.1.** The Durer graph is a prime cordial graph.

**Proof.**

Let  $G$  be the Durer graph. Let  $V(G)=\{v_1,v_2,\dots,v_{12}\}$  and  $E(G)=\{v_i v_{i+1} : 1\leq i\leq 11\} \cup \{v_1 v_{12}, v_1 v_9, v_2 v_7, v_3 v_5, v_4 v_{12}, v_6 v_{11}, v_8 v_{10}\}$ . Then  $|V(G)|=12$  and  $|E(G)|=18$ .

We define a vertex labeling map  $f:V(G)\rightarrow\{1,2,3,\dots,12\}$  as follows:

$f(v_1)=10, f(v_2)=4, f(v_3)=6, f(v_4)=8, f(v_5)=2, f(v_6)=12, f(v_7)=1, f(v_8)=7, f(v_9)=5, f(v_{10})=3, f(v_{11})=9, f(v_{12})=11$ .

The labels of the vertices of the Durer graph are shown in Figure 1.

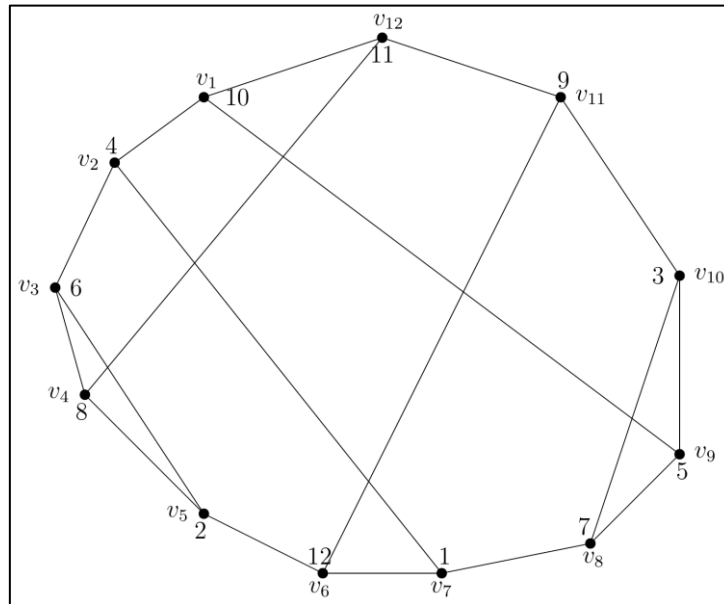


Figure 1. The prime cordial labeling of the Durer graph.

From the above defined vertex labeling map, we observe that  $e_f(0)=e_f(1)=9$ . Thus  $|e_f(0)-e_f(1)|=0$ . Hence the Durer graph admits prime cordial labeling (Definition 2.7).  $\square$

**Theorem 3.2.** *The Heawood graph is a prime cordial graph.*

**Proof.**

Let  $G$  be the Heawood graph. Let  $V(G)=\{v_1, v_2, \dots, v_{14}\}$  and  $E(G)=\{v_i v_{i+1} : 1 \leq i \leq 13\} \cup \{v_{2i+1} v_{2i+6} : 0 \leq i \leq 4\} \cup \{v_1 v_{14}, v_2 v_{11}, v_4 v_{13}\}$ . Then  $|V(G)|=14$  and  $|E(G)|=21$ .

We define  $f:V(G) \rightarrow \{1, 2, \dots, 14\}$  as:  $f(v_1)=1, f(v_2)=9, f(v_3)=12, f(v_4)=10, f(v_5)=13, f(v_6)=11, f(v_7)=14, f(v_8)=8, f(v_9)=6, f(v_{10})=4, f(v_{11})=2, f(v_{12})=7, f(v_{13})=5, f(v_{14})=3$ .

The labels of the vertices of the Heawood graph are shown in Figure 2.

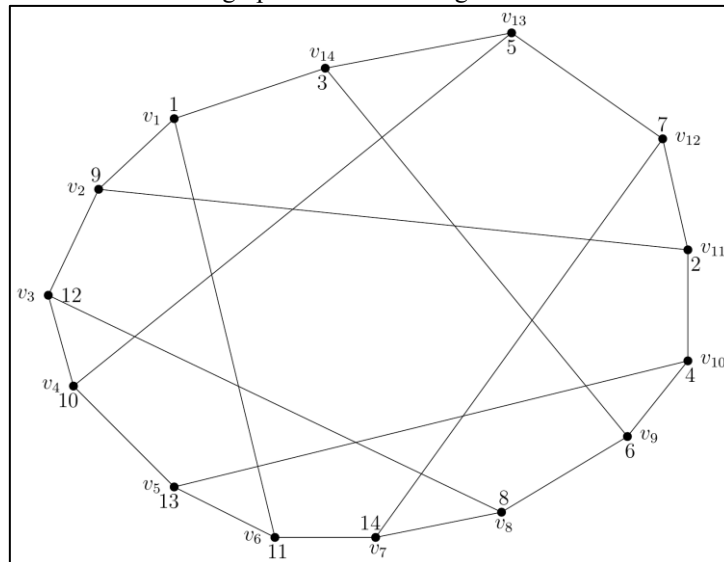


Figure 2. The prime cordial labeling of the Heawood graph.

We observe that  $e_f(0)=10, e_f(1)=11$ . Thus  $|e_f(0)-e_f(1)|=1$ . Hence the Heawood graph is a prime cordial graph.  $\square$

**Theorem 3.3.** *The Frucht graph is a prime cordial graph.*

**Proof.**

Let  $G$  be the Frucht graph. Let  $V(G)=\{v_1, v_2, \dots, v_{12}\}$  and  $E(G)=\{v_i v_{i+1} : 1 \leq i \leq 6\} \cup \{v_1 v_7, v_1 v_8, v_2 v_8, v_8 v_{11}, v_3 v_9, v_4 v_9, v_9 v_{12}, v_5 v_{10}, v_6 v_{10}, v_{10} v_{12}, v_7 v_{11}, v_{11} v_{12}\}$ . Then  $|V(G)|=12$  and  $|E(G)|=18$ .

$f(v_1)=1, f(v_2)=11, f(v_3)=7, f(v_4)=5, f(v_5)=4, f(v_6)=2, f(v_7)=3, f(v_8)=9, f(v_9)=10, f(v_{10})=8, f(v_{11})=6, f(v_{12})=12$ .

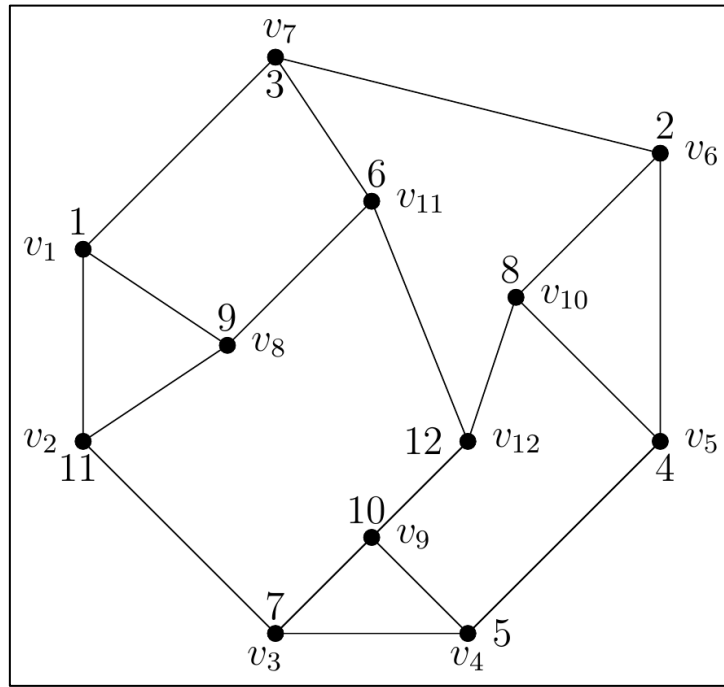


Figure 3. The prime cordial labeling of the Frucht graph.

We observe that  $e_f(0)=e_f(1)=9$ . Thus  $|e_f(0)-e_f(1)|=0$ . Hence the Frucht graph is a prime cordial graph.  $\square$

**Theorem 3.4.** *The Tietze graph is a prime cordial graph.*

**Proof.**

Let  $G$  be the Tietze graph. Let  $V(G)=\{v_1, v_2, \dots, v_{12}\}$  and  $E(G)=\{v_i v_{i+1} : 1 \leq i \leq 8\} \cup \{v_1 v_9, v_{10} v_1, v_{10} v_{11}, v_{10} v_{12}, v_4 v_{11}, v_{11} v_{12}, v_7 v_{12}, v_2 v_6, v_3 v_8, v_5 v_9\}$ . Then  $|V(G)|=12$  and  $|E(G)|=18$ .

$f(v_1)=4, f(v_2)=2, f(v_3)=12, f(v_4)=11, f(v_5)=9, f(v_6)=7, f(v_7)=5, f(v_8)=3, f(v_9)=6, f(v_{10})=8, f(v_{11})=1, f(v_{12})=10$ .

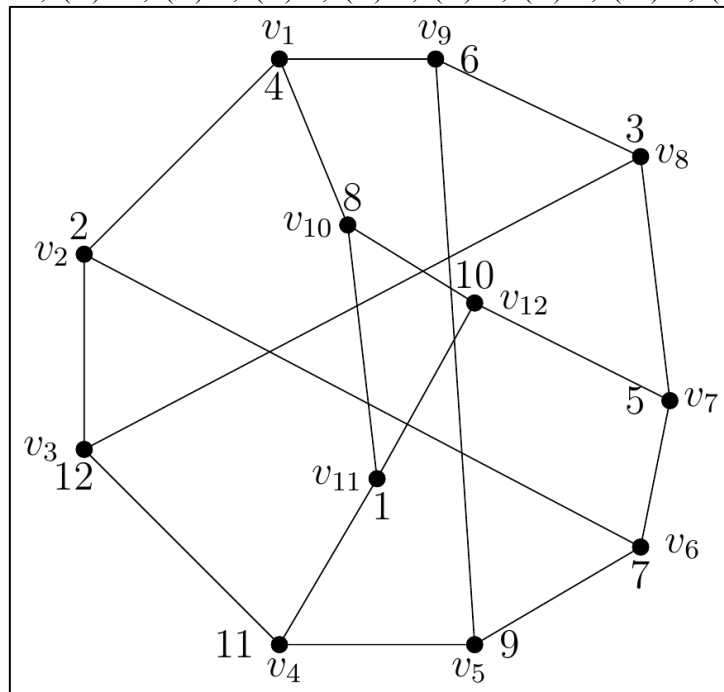


Figure 4. The prime cordial labeling of the Tietze graph.

We observe that  $e_f(0)=e_f(1)=9$ , so  $|e_f(0)-e_f(1)|=0$ . Hence the Tietze graph is a prime cordial graph.  $\square$

**Theorem 3.5.** *The hypohamiltonian graph is a prime cordial graph.*

**Proof.**

Let  $\{v_1, v_2, \dots, v_{16}\}$  be the vertices of the hypohamiltonian graph  $G$  with apex vertex  $v_{16}$  and  $E(G)=\{v_i v_{i+1} : 1 \leq i \leq 14\} \cup \{v_{16} v_{3+i} : 0 \leq i \leq 4\} \cup \{v_{3+i} v_{3+i+6} : 0 \leq i \leq 3\} \cup \{v_1 v_{15}, v_{14} v_3\}$ . Then  $|V(G)|=16$  and  $|E(G)|=25$ .

$f(v_1)=12, f(v_2)=15, f(v_3)=13, f(v_4)=11, f(v_5)=9, f(v_6)=7, f(v_7)=5, f(v_8)=3, f(v_9)=1, f(v_{10})=2, f(v_{11})=4, f(v_{12})=6, f(v_{13})=8, f(v_{14})=10, f(v_{15})=14, f(v_{16})=16$ .

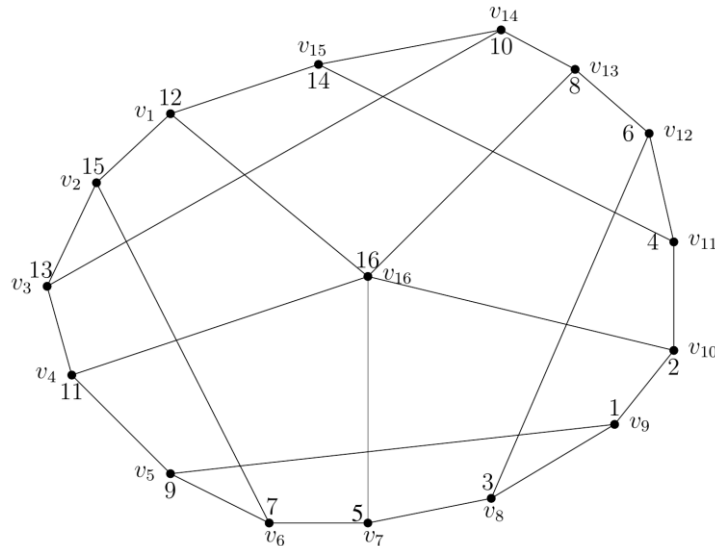


Figure 5. The prime cordial labeling of the hypohamiltonian graph.

We observe that  $e_f(0)=12, e_f(1)=13$ , so  $|e_f(0)-e_f(1)|=1$ . Hence the hypohamiltonian graph  $G$  is a prime cordial graph.  $\square$

**Theorem 3.6.** *The cubic graph with 12 vertices is a prime cordial graph.*

**Proof.**

Let  $G$  be the cubic graph with 12 vertices. Let  $V(G)=\{v_1, \dots, v_{12}\}$  and  $E(G)=\{v_i v_{i+1} : 1 \leq i \leq 5\} \cup \{v_i v_{i+1} : 7 \leq i \leq 11\} \cup \{v_1 v_6, v_7 v_{12}, v_1 v_7, v_2 v_8, v_3 v_5, v_4 v_{10}, v_5 v_9, v_6 v_{11}\}$ . Then  $|V(G)|=12$  and  $|E(G)|=18$ .

$f(v_i)=2i, 1 \leq i \leq 6; f(v_7)=1, f(v_8)=11, f(v_9)=9, f(v_{10})=7, f(v_{11})=5, f(v_{12})=3$ .

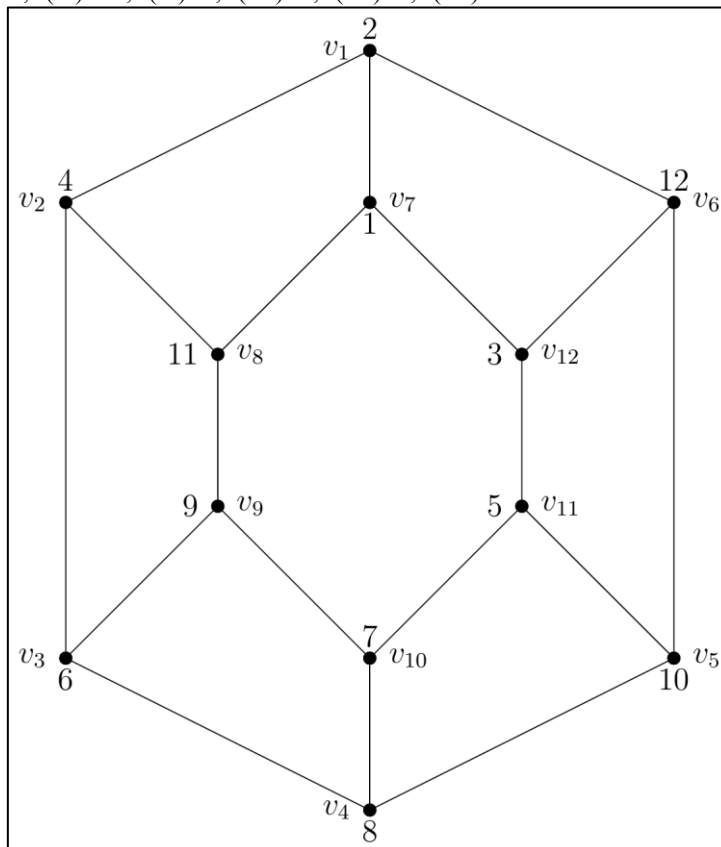


Figure 6. The prime cordial labeling of the cubic graph with 12 vertices.

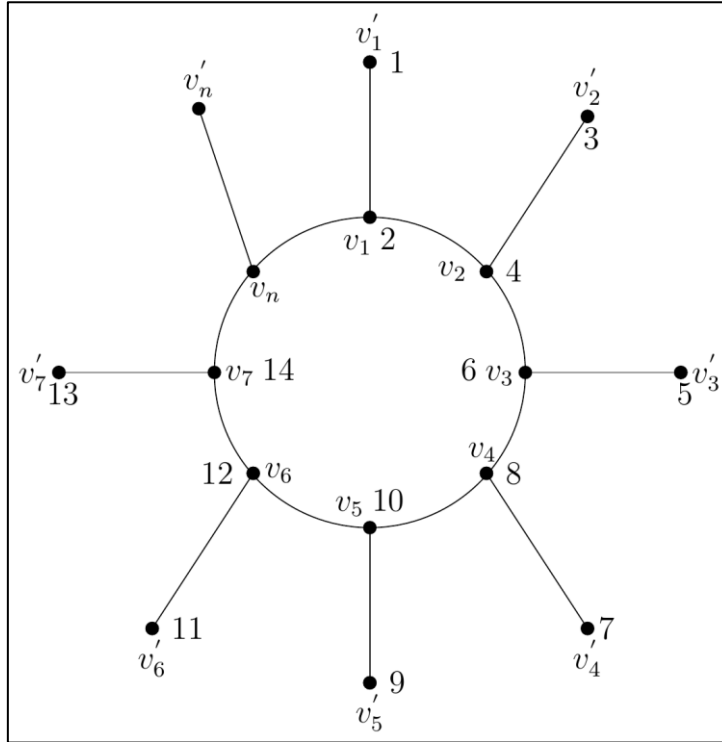
We observe that  $e_f(0)=e_f(1)=9$ , so  $|e_f(0)-e_f(1)|=0$ . Hence the cubic graph with 12 vertices admits prime cordial labeling.  $\square$

**Theorem 3.7.** *The crown graph  $C_n^+$  is a prime cordial graph.*

**Proof.**

Let  $C_n^+$  be the crown graph with  $2n$  vertices. Let  $V(C_n^+)=\{v_1, \dots, v_n, v_1', \dots, v_n'\}$  and  $E(C_n^+)=\{v_i v_i' : 1 \leq i \leq n\} \cup \{v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_i v_n\}$ . Then  $|V(C_n^+)|=2n$  and  $|E(C_n^+)|=2n$ .

$f(v_i)=2i, 1 \leq i \leq n; f(v'_i)=2i-1, 1 \leq i \leq n.$



**Figure 7.** The prime cordial labeling of the crown graph  $C_n^+$ .

We observe that  $e_f(0)=e_f(1)=n$ , so  $|e_f(0)-e_f(1)|=0$ . Hence the crown graph  $C_n^+$  is a prime cordial graph.  $\square$

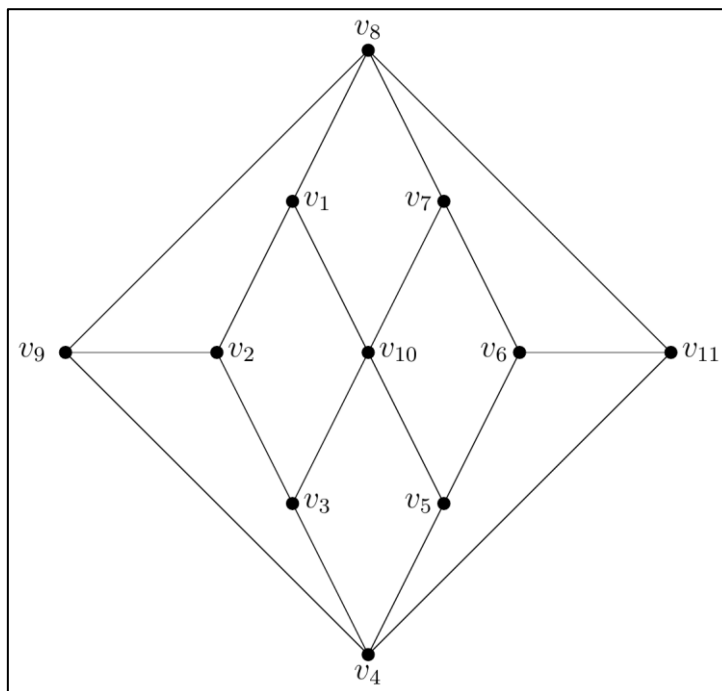
**4. Some New Results: Non-Prime-Cordial Graphs**

In this section, we prove that the Herschel graph  $H_s$ , truncated tetrahedron graph, Moser spindle graph, and Wagner graph are not prime cordial graphs.

**Theorem 4.1.** *The Herschel graph  $H_s$  is not a prime cordial graph.*

**Proof.**

Let  $\{v_1, v_2, \dots, v_{11}\}$  be the vertices of the Herschel graph  $H_s$  with central vertex  $v_{10}$  and  $E(H_s)=\{v_i v_{i+1} : 1 \leq i \leq 7\} \cup \{v_{10} v_{2i+1} : 0 \leq i \leq 3\} \cup \{v_1 v_8, v_9 v_2, v_9 v_4, v_9 v_8, v_1 v_4, v_{11} v_6, v_{11} v_8\}$ . Then  $|V(H_s)|=11$  and  $|E(H_s)|=18$ .



**Figure 8.** The Herschel graph  $H_s$ .

In graph  $H_s$ , only the pairs  $(2,4),(2,6),(2,8),(2,10),(3,6),(3,9),(4,6),(4,8),(5,10),(6,8),(6,10),(8,10)$  yield edge label 0. Any labeling contains at most eight such pairs, so  $e_f(0) \leq 8$  and  $e_f(1) \geq 10$ . Thus  $|e_f(0) - e_f(1)| \geq 2$ . Hence  $H_s$  is not a prime cordial graph.  $\square$

**Theorem 4.2.** *The Moser spindle graph is not a prime cordial graph.*

**Proof.**

Let  $G$  be the Moser spindle graph. Let  $V(G) = \{v_1, \dots, v_7\}$  and  $E(G) = \{v_i v_{i+1} : 1 \leq i \leq 4\} \cup \{v_1 v_5, v_1 v_6, v_2 v_6, v_5 v_6, v_3 v_7, v_4 v_7, v_5 v_7\}$ . Then  $|V(G)| = 7$  and  $|E(G)| = 11$ .

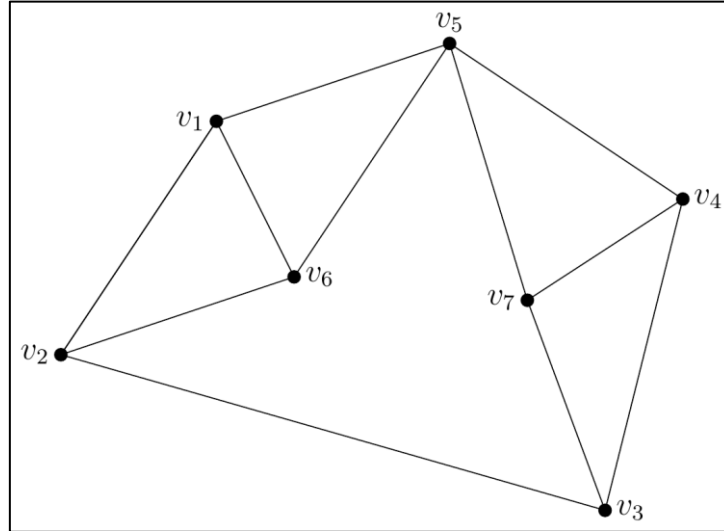


Figure 9. The Moser spindle graph.

Only pairs  $(2,4),(2,6),(3,6),(4,6)$  yield edge label 0. Hence  $e_f(0) \leq 4$  and  $e_f(1) \geq 7$ , giving  $|e_f(0) - e_f(1)| \geq 3$ . Hence the Moser spindle graph is not a prime cordial graph.  $\square$

**Theorem 4.3.** *The Wagner graph is not a prime cordial graph.*

**Proof.**

Let  $G$  be the Wagner graph. Let  $V(G) = \{v_1, \dots, v_8\}$  and  $E(G) = \{v_i v_{i+1} : 1 \leq i \leq 7\} \cup \{v_1 v_8, v_1 v_5, v_2 v_6, v_3 v_7, v_4 v_8\}$ . Then  $|V(G)| = 8$  and  $|E(G)| = 12$ .

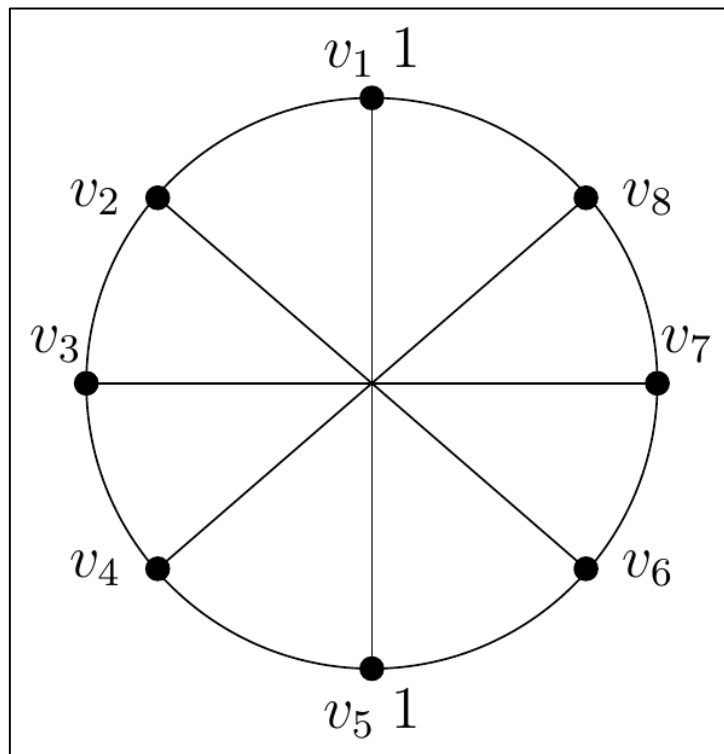


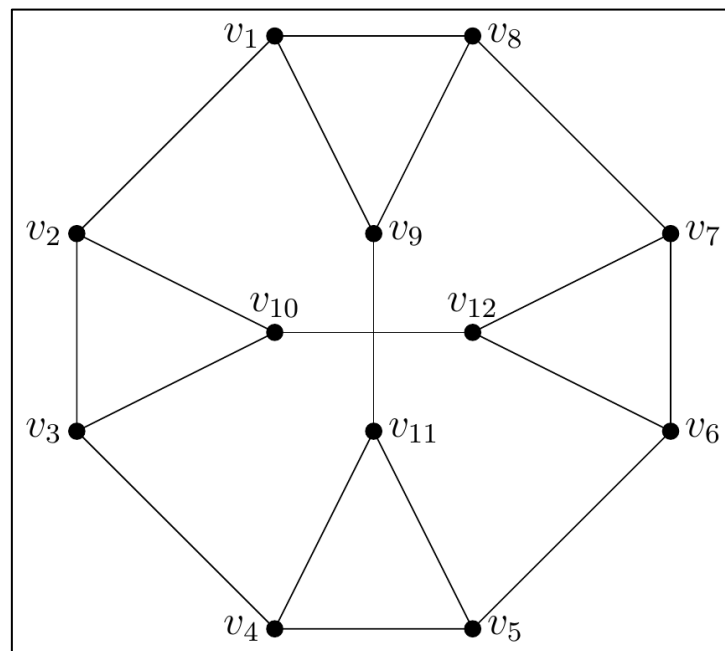
Figure 10. The Wagner graph.

Only pairs  $(2,4),(2,6),(2,8),(3,6),(4,6),(4,8),(6,8)$  yield edge label 0. Hence  $e_f(0) \leq 4$  and  $e_f(1) \geq 8$ , so  $|e_f(0) - e_f(1)| \geq 4$ . Hence the Wagner graph is not a prime cordial graph.  $\square$

**Theorem 4.4.** *The truncated tetrahedron graph is not a prime cordial graph.*

**Proof.**

Let  $G$  be the truncated tetrahedron graph. Let  $V(G)=\{v_1, \dots, v_{12}\}$  and  $E(G)=\{v_i v_{i+1} : 1 \leq i \leq 7\} \cup \{v_1 v_8, v_9 v_1, v_9 v_8, v_9 v_{11}, v_{10} v_2, v_{10} v_3, v_{10} v_{12}, v_{11} v_4, v_{11} v_5, v_{12} v_6, v_{12} v_7\}$ . The graph  $G$  has 12 vertices and 18 edges.



**Figure 11.** The truncated tetrahedron graph.

Only the pairs

$(2,4), (2,6), (2,8), (2,10), (2,12), (3,6), (3,9), (3,12), (4,6), (4,8), (4,12), (5,10), (5,12), (6,8), (6,10), (6,12), (8,10), (8,12), (10,12)$  yield edge label 0. Any labeling contains at most eight such pairs, so  $e_f(0) \leq 8$  and  $e_f(1) \geq 10$ . Thus  $|e_f(0) - e_f(1)| \geq 2$ . Hence the truncated tetrahedron graph is not a prime cordial graph.  $\square$

## 5. Application: Prime Cordial Labeling in Epidemic Spread Networks

The prime cordial labeling framework introduced in the preceding sections — with its GCD-based edge-assignment rule (Definition 2.7) and near-balanced partition of edges into classes 0 and 1 — has direct applications in the mathematical modeling of epidemic disease spread. In this section we develop a three-part application that maps the results of Theorems 3.1–3.7 and Theorems 4.1–4.4 onto an epidemic contact network framework.

### 5.1 Contact Network Model and Prime Cordial Interpretation

In mathematical epidemiology, a contact network is a graph  $G=(V,E)$  in which each vertex represents an individual or sub-population and each edge represents a potential disease transmission pathway. A prime cordial labeling  $f:V(G) \rightarrow \{1,2,\dots,p\}$  assigns each individual a unique immune status index. The induced edge label classifies every contact pathway:  $f^*(uv)=1$  if  $\gcd(f(u),f(v))=1$  (active transmission), and  $f^*(uv)=0$  if  $\gcd(f(u),f(v))>1$  (inactive/blocked).

The prime cordial condition  $|e_f(0) - e_f(1)| \leq 1$  expresses a near-perfect balance between active and inactive transmission pathways, which has a direct implication for the basic reproduction number  $R_0$ . Graphs that do not admit prime cordial labeling (Theorems 4.1–4.4) model contact networks where no immune status assignment can achieve near-equal active and inactive pathways — the Herschel ( $|e_f(0) - e_f(1)| \geq 2$ ), Moser spindle ( $\geq 3$ ), Wagner ( $\geq 4$ ), and truncated tetrahedron ( $\geq 2$ ) represent structurally resistant epidemic networks.

### 5.2 SIR Dynamics on Prime Cordial Networks

We embed the standard SIR model onto the contact network  $G$ . The transmission rate along edge  $uv$  is weighted by  $\beta_{uv} = \beta_0 \cdot f^*(uv)$ . The effective reproduction number for vertex  $v$  is  $R_v = (\beta_0/\gamma) \cdot \deg_1(v)$ , where  $\deg_1(v)$  counts active (label-1) pathway neighbors. The network basic reproduction number is  $R_0 = (\beta_0/\gamma) \cdot \max_v \deg_1(v)$ . For the crown graph  $C_n^+$ ,  $R_0(C_n^+) = \beta_0/\gamma$ , independent of  $n$ , making it a canonical model for hub-and-spoke epidemic containment.

### 5.3 Epidemic Data Encoding via Prime Cordial Labeling

The prime cordial labeling framework provides a mechanism for secure encoding of epidemic surveillance data. Since the prime cordial condition ensures  $|e_f(0) - e_f(1)| \leq 1$ , the encoded edge-label string has nearly equal numbers of 0s and 1s, providing maximum entropy and resistance to frequency-analysis attacks. Non-prime-cordial graphs cannot serve as codebooks since any encoding attempt is detectable via imbalance analysis.

**Table 1.** Prime cordial epidemic parameters for graphs in Theorems 3.1–3.7.

Graph	p	q	e_f(0)	e_f(1)	e_f(0)–e_f(1)	Epidemic Status
Durer (Thm 3.1)	12	18	9	9	0	Balanced; controllable
Heawood (Thm 3.2)	14	21	10	11	1	Near-balanced; controllable
Frucht (Thm 3.3)	12	18	9	9	0	Balanced; controllable
Tietze (Thm 3.4)	12	18	9	9	0	Balanced; controllable
Hypohamiltonian (Thm 3.5)	16	25	12	13	1	Near-balanced; controllable
Cubic-12 (Thm 3.6)	12	18	9	9	0	Balanced; controllable
Crown $C_n^+$ (Thm 3.7)	2n	2n	n	n	0	Perfectly balanced
Herschel (Thm 4.1)	11	18	$\leq 8$	$\geq 10$	$\geq 2$	Structurally resistant
Moser spindle (Thm 4.2)	7	11	$\leq 4$	$\geq 7$	$\geq 3$	Highly resistant
Wagner (Thm 4.3)	8	12	$\leq 4$	$\geq 8$	$\geq 4$	Most resistant
Truncated tetrahedron (Thm 4.4)	12	18	$\leq 8$	$\geq 10$	$\geq 2$	Structurally resistant

**Table 2.** Summary: Prime cordial epidemic network and encoding properties.

Graph	Theorem	Epidemic Network Property	Encoding Suitability
Durer	Thm 3.1	Perfectly balanced; $R_0$ halved	Secure codebook
Heawood	Thm 3.2	Near-balanced; controllable	Secure codebook
Frucht	Thm 3.3	Perfectly balanced	Secure codebook
Tietze	Thm 3.4	Perfectly balanced	Secure codebook
Hypohamiltonian	Thm 3.5	Near-balanced; scalable	Secure codebook
Cubic-12	Thm 3.6	Perfectly balanced	Secure codebook
Crown $C_n^+$	Thm 3.7	Hub-spoke; scale-free $R_0$	Canonical scalable model
Herschel $H_s$	Thm 4.1	Structurally resistant	Unsuitable (imbalanced)
Moser spindle	Thm 4.2	Highly resistant	Unsuitable (imbalanced)
Wagner	Thm 4.3	Most resistant; $\Delta \geq 4$	Unsuitable (imbalanced)
Truncated tetrahedron	Thm 4.4	Structurally resistant	Unsuitable (imbalanced)

### 6. Concluding Remarks

In the present work we investigated prime cordial labeling for seven families of well-known graphs and proved four non-admissibility results. The graphs Durer, Heawood, Frucht, Tietze, hypohamiltonian, cubic (12 vertices), and crown  $C_n^+$  are prime cordial (Theorems 3.1–3.7), while the Herschel, Moser spindle, Wagner, and truncated tetrahedron graphs are not (Theorems 4.1–4.4). Furthermore, as demonstrated in Section 5, the prime cordial labeling framework provides a rigorous mathematical basis for epidemic network analysis, hub-and-spoke epidemic modeling with scale-free reproduction number, and secure epidemic surveillance data encoding.

## Acknowledgement

The authors gratefully acknowledge the financial support provided by Sacred Heart College through the DB Grant (SHC/DB Grant/2025-2026/07). This research would not have been possible without the institutional support and resources made available through this funding.

## References

- [1] Abdullah Aljouiee. On prime cordial labeling of graphs. *Kyungpook Mathematical Journal*, 56(1):41–46, 2016.
- [2] J. Baskar Babujee and L. Shobana. Prime and prime cordial labeling for some special graphs. *International Journal of Contemporary Mathematical Sciences*, 2010.
- [3] I. Cahit. Cordial graphs: A weaker version of graceful and harmonious graphs. *Ars Combinatoria*, 23:201–207, 1987.
- [4] Solomon W. Golomb. How to number a graph. In *Graph Theory and Computing*, pages 23–37. Academic Press, New York, 1972.
- [5] Ronald L. Graham and Neil J. A. Sloane. On additive bases and harmonious graphs. *SIAM Journal on Algebraic and Discrete Methods*, 1:382–404, 1980.
- [6] Frank Harary. *Graph Theory*. Addison-Wesley, Reading, Massachusetts, 1972.
- [7] Q. D. Kang, Z. H. Liang, Y. Z. Gao, and G. H. Yang. On labeling of some graphs. *Journal of Combinatorial Mathematics and Combinatorial Computing*, 22:193–210, 1996.
- [8] S. Lo. On edge graceful labelings of graphs. *Congressus Numerantium*, 50:231–241, 1985.
- [9] Muhammad Naeem, Sehrsiah Waseem, Shahid Qaisar, Junaid Ahmad, and Muhammad Kamran Siddiqui. On prime cordial labeling to pattern recognition of knight graphs. *Discrete Mathematics, Algorithms and Applications*, 2025. Article No. 2550172.
- [10] A. Parthiban and Vishally Sharma. A comprehensive survey on prime cordial and divisor cordial labeling of graphs. *Journal of Physics: Conference Series*, 1531(1):012074, 2020.
- [11] Alexander Rosa. On certain valuations of the vertices of a graph. In *Theory of Graphs (Rome, 1966)*, pages 349–355. Gordon and Breach, 1967.
- [12] A. Sugumaran and V. Mohan. Further results on prime cordial labeling. *Annals of Pure and Applied Mathematics*, 14(3):489–496, 2017.
- [13] A. Sugumaran and P. V. Prakash. Prime cordial labeling for theta graph. *Annals of Pure and Applied Mathematics*, 14(3):379–386, 2017.
- [14] M. Sundaram, R. Ponraj, and S. Somasundaram. Prime cordial labeling of graphs. *Journal of the Indian Academy of Mathematics*, 27:373–390, 2005.
- [15] S. K. Vaidya and N. H. Shah. Some new results on prime cordial labeling. *ISRN Combinatorics*, 2014.