Prime Cordial Labeling of Contact Networks: An Environmental Application to Epidemic Spread in Aquatic Populations

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Abstract

A **prime cordial labeling** of a graph G is a bijection f:V(G)→{1,2,…,|V(G)|} such that each edge uv∈E(G) is assigned label 1 if gcd(f(u),f(v))=1 and 0 if gcd(f(u),f(v))>1; further the number of edges labeled 0 and 1 differ by at most 1. If a graph admits prime cordial labeling, then it is called a prime cordial graph. We prove that the Durer graph, Heawood graph, Frucht graph, Tietze graph, hypohamiltonian graph, cubic graph with 12 vertices, and crown graph are prime cordial graphs. Also we prove that the Herschel graph, Wagner graph, Moser spindle graph, and truncated tetrahedron graph are not prime cordial graphs.

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Introduction

Prime cordial labeling is an important variation of cordial labeling in graph theory, combining ideas from graph labeling and elementary number theory. The concept of cordial labeling was introduced by Cahit [3] as a weaker alternative to graceful and harmonious labeling. Building on this idea, Sundaram, Ponraj, and Somasundaram [14] introduced prime cordial labeling, where vertices are assigned labels so that the induced edge labels, determined by relative primality, satisfy cordial balance conditions.

Since its introduction, several researchers have investigated prime cordial labeling for different families of graphs. Aljouiee [1] studied structural properties and provided results for various graph classes. Babujee and Shobana [2] examined prime and prime cordial labelings for selected special graphs. Kang et al. [7] contributed earlier results on labeling techniques. Sugumaran and Mohan [12], and Sugumaran and Prakash [13], extended results to theta graphs and related families. Parthiban and Sharma [10] provided a survey on prime cordial and divisor cordial labeling. Naeem et al. [9] explored applications in pattern recognition of knight graphs.

Overall, the existing literature demonstrates steady progress in characterizing graph classes that admit prime cordial labeling. Despite these developments, many graph families remain unexplored. These include A-cordial labeling, product cordial labeling, total product cordial labeling, divisor cordial labeling, sum divisor cordial labeling, and prime cordial labeling, among others. First we recall the basic definitions of labeling.

Preliminaries

Definition 2.1 (Labeling) [6]**.** A labeling of a graph G is an assignment of integers to the vertices or edges or both.

Definition 2.2 (Binary Vertex Labeling) [3]**.** Let G be a graph and a labeling map h assigns each vertex w of G as either 0 or 1. Then the graph is called a binary vertex labeling under h. The induced edge labeling is the absolute difference of labels of end vertices under $h^*:E(G)\rightarrow\{0,1\}$.

We denote $w_h(k)$ and $e_h(k)$ as the number of vertices and edges of G having label k under h^*, where $k=0,1$.

Definition 2.3 (β-Valuation) [11]**.** A β-valuation of a graph G with q edges is an injection h from vertices of G to $\{0,1,2,\ldots,q\}$ such that each edge $e=uw$ is assigned the label $|h(u)-h(w)|$ and the resulting edge labels are distinct.

Definition 2.4 (Graceful Labeling) [4]**.** Let h be an injection from vertices of G to $\{0,1,2,\ldots,q\}$. This is a graceful labeling of G if we assign each edge uw the label $|h(u)-h(w)|$ and the resulting edge labels are distinct.

Definition 2.5 (Harmonious Labeling) [5]**.** The harmonious labeling of a graph G with q edges is an injection h from vertices to $\{0,1,2,\ldots,q-1\}$ such that each edge $e=uw$ is assigned $(h(u)+h(w)) \bmod q$ and the resulting edge labels are distinct.

Definition 2.6 (Cordial Labeling) [3]**.** A binary vertex labeling of a given graph G is a cordial labeling if each edge $e=uw$ is assigned $|h(u)-h(w)|$, the number of vertices having labels 0 and 1 differ by at most 1, and the number of edges having labels 0 and 1 differ by at most 1. If G admits cordial labeling then it is called a cordial graph.

Definition 2.7 (Prime Cordial Labeling) [14]**.** A prime cordial labeling of a graph G is a bijection $f:V(G)\rightarrow\{1,2,\ldots,|V(G)|\}$ such that each edge $uv\in E(G)$ is assigned label 1 if $\gcd(f(u),f(v))=1$ and 0 if $\gcd(f(u),f(v))>1$; further the number of edges labeled 0 and 1 differ by at most 1. If a graph admits prime cordial labeling, then it is called a prime cordial graph.

Main Results

We investigate the prime cordial labeling concept for the Durer graph, Heawood graph, Frucht graph, Tietze graph, hypohamiltonian graph, cubic graph with 12 vertices, and crown graph. Further we show that some graphs do not admit prime cordial labeling.

Theorem 3.1. The Durer graph is a prime cordial graph.

Proof.

Let G be the Durer graph. Let $V(G)=\{v_1,v_2,\ldots,v_{12}\}$ and $E(G)=\{v_iv_{i+1} : 1\leq i\leq 11\} \cup \{v_1v_{12}, v_1v_9, v_2v_7, v_3v_5, v_4v_{12}, v_6v_{11}, v_8v_{10}\}$. Then $|V(G)|=12$ and $|E(G)|=18$.

We define a vertex labeling map $f:V(G)\rightarrow\{1,2,3,\ldots,12\}$ as follows:

$f(v_1)=10$, $f(v_2)=4$, $f(v_3)=6$, $f(v_4)=8$, $f(v_5)=2$, $f(v_6)=12$, $f(v_7)=1$, $f(v_8)=7$, $f(v_9)=5$, $f(v_{10})=3$, $f(v_{11})=9$, $f(v_{12})=11$.

The labels of the vertices of the Durer graph are shown in Figure 1.

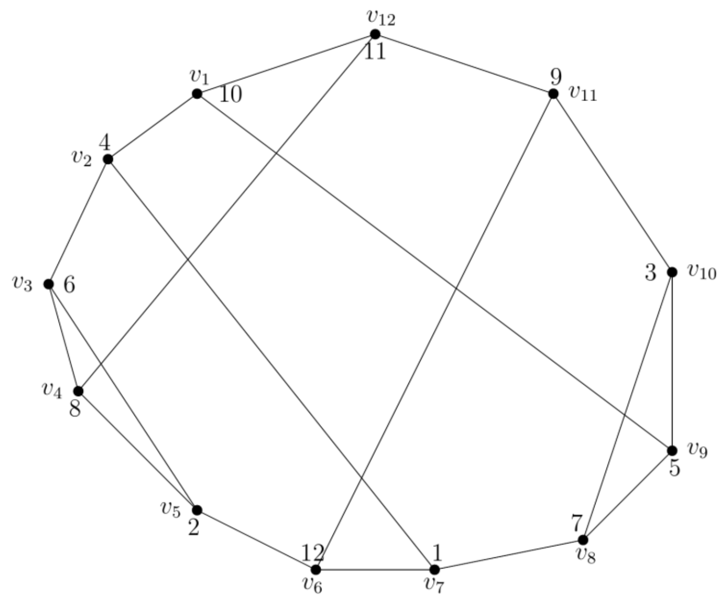

Figure 1. The prime cordial labeling of the Durer graph.

From the above defined vertex labeling map, we observe that $e\_f(0)=e\_f(1)=9$. Thus $|e\_f(0)-e\_f(1)|=0$. Hence the Durer graph admits prime cordial labeling (Definition 2.7). □

Theorem 3.2. The Heawood graph is a prime cordial graph.

Proof.

Let G be the Heawood graph. Let $V(G)=\{v_1,v_2,\dots,v_{14}\}$ and $E(G)=\{v_iv_{i+1} : 1\leq i\leq 13\} \cup \{v_{2i+1}v_{2i+6} : 0\leq i\leq 4\} \cup \{v_1v_{14}, v_2v_{11}, v_4v_{13}\}$. Then $|V(G)|=14$ and $|E(G)|=21$.

We define $f:V(G)\rightarrow\{1,2,\dots,14\}$ as: $f(v_1)=1$, $f(v_2)=9$, $f(v_3)=12$, $f(v_4)=10$, $f(v_5)=13$, $f(v_6)=11$, $f(v_7)=14$, $f(v_8)=8$, $f(v_9)=6$, $f(v_{10})=4$, $f(v_{11})=2$, $f(v_{12})=7$, $f(v_{13})=5$, $f(v_{14})=3$.

The labels of the vertices of the Heawood graph are shown in Figure 2.

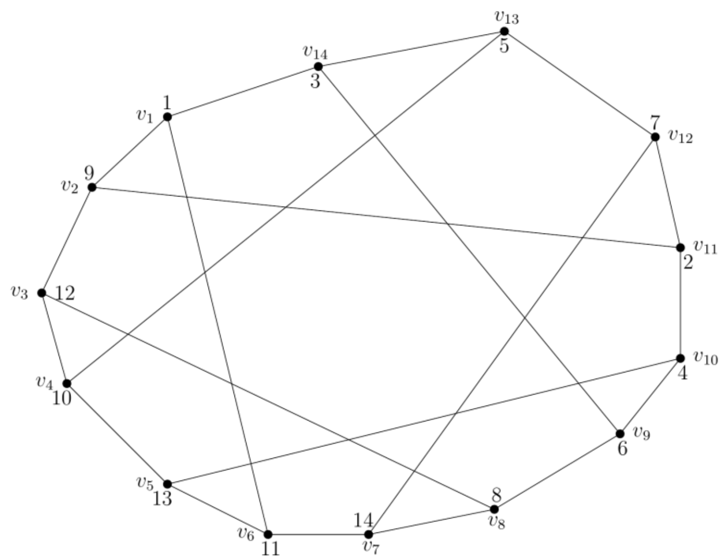

Figure 2. The prime cordial labeling of the Heawood graph.

We observe that $e\_f(0)=10$, $e\_f(1)=11$. Thus $|e\_f(0)-e\_f(1)|=1$. Hence the Heawood graph is a prime cordial graph. □

Theorem 3.3. The Frucht graph is a prime cordial graph.

Proof.

Let G be the Frucht graph. Let $V(G)=\{v_1,v_2,\dots,v_{12}\}$ and $E(G)=\{v_iv_{i+1} : 1\leq i\leq 6\} \cup \{v_1v_7, v_1v_8, v_2v_8, v_8v_{11}, v_3v_9, v_4v_9, v_9v_{12}, v_5v_{10}, v_6v_{10}, v_{10}v_{12}, v_7v_{11}, v_{11}v_{12}\}$. Then $|V(G)|=12$ and $|E(G)|=18$.

$f(v_1)=1$, $f(v_2)=11$, $f(v_3)=7$, $f(v_4)=5$, $f(v_5)=4$, $f(v_6)=2$, $f(v_7)=3$, $f(v_8)=9$, $f(v_9)=10$, $f(v_{10})=8$, $f(v_{11})=6$, $f(v_{12})=12$.

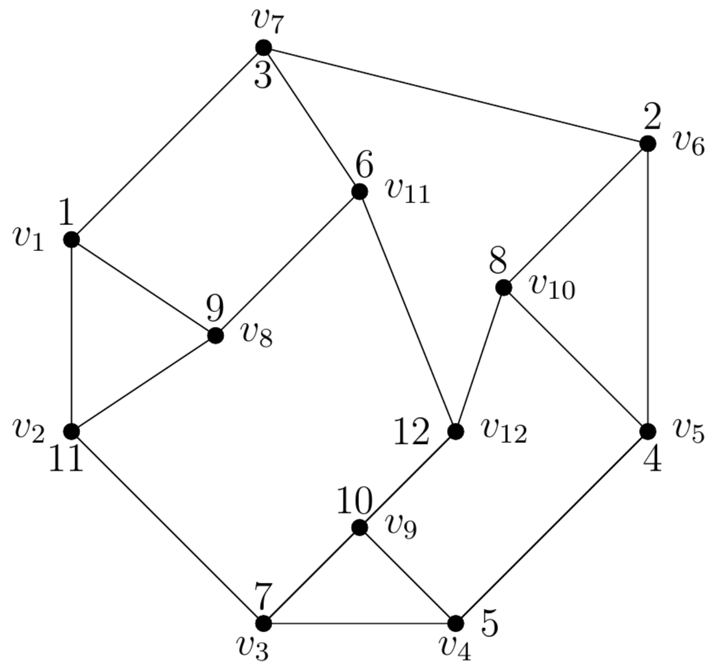

Figure 3. The prime cordial labeling of the Frucht graph.

We observe that $e_f(0)=e_f(1)=9$. Thus $|e_f(0)-e_f(1)|=0$. Hence the Frucht graph is a prime cordial graph. □

Theorem 3.4. The Tietze graph is a prime cordial graph.

Proof.

Let G be the Tietze graph. Let $V(G)=\{v_1,v_2,\ldots,v_{12}\}$ and $E(G)=\{v_iv_{i+1} : 1\leq i\leq 8\} \cup \{v_1v_9, v_{10}v_1, v_{10}v_{11}, v_{10}v_{12}, v_4v_{11}, v_{11}v_{12}, v_7v_{12}, v_2v_6, v_3v_8, v_5v_9\}$. Then $|V(G)|=12$ and $|E(G)|=18$.

$f(v_1)=4$, $f(v_2)=2$, $f(v_3)=12$, $f(v_4)=11$, $f(v_5)=9$, $f(v_6)=7$, $f(v_7)=5$, $f(v_8)=3$, $f(v_9)=6$, $f(v_{10})=8$, $f(v_{11})=1$, $f(v_{12})=10$.

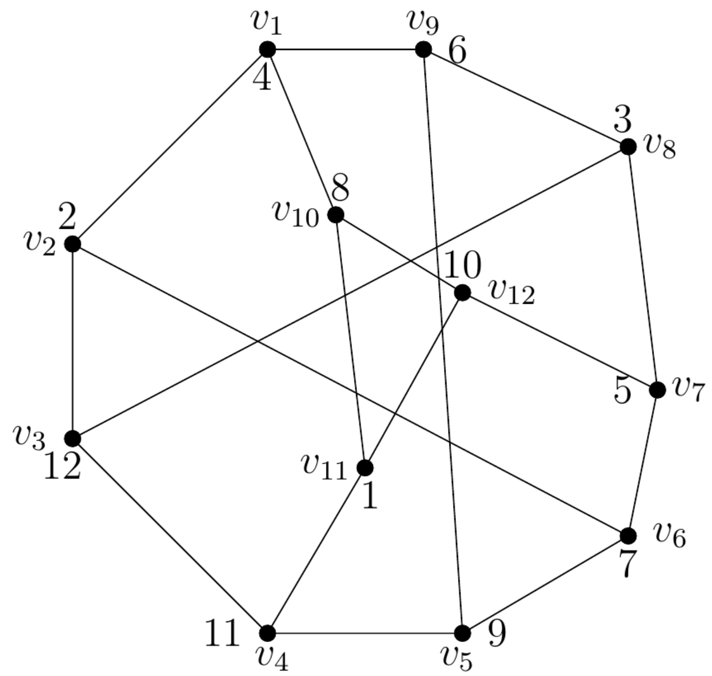

Figure 4. The prime cordial labeling of the Tietze graph.

We observe that $e_f(0)=e_f(1)=9$, so $|e_f(0)-e_f(1)|=0$. Hence the Tietze graph is a prime cordial graph. □

Theorem 3.5. The hypohamiltonian graph is a prime cordial graph.

Proof.

Let $\{v_1,v_2,\ldots,v_{16}\}$ be the vertices of the hypohamiltonian graph G with apex vertex v_{16} and $E(G)=\{v_iv_{i+1} : 1\le i\le 14\} \cup \{v_{16}v_{3i+1} : 0\le i\le 4\} \cup \{v_{3i+2}v_{3i+6} : 0\le i\le 3\} \cup \{v_1v_{15}, v_{14}v_3\}$. Then $|V(G)|=16$ and $|E(G)|=25$.

$f(v_1)=12$, $f(v_2)=15$, $f(v_3)=13$, $f(v_4)=11$, $f(v_5)=9$, $f(v_6)=7$, $f(v_7)=5$, $f(v_8)=3$, $f(v_9)=1$, $f(v_{10})=2$, $f(v_{11})=4$, $f(v_{12})=6$, $f(v_{13})=8$, $f(v_{14})=10$, $f(v_{15})=14$, $f(v_{16})=16$.

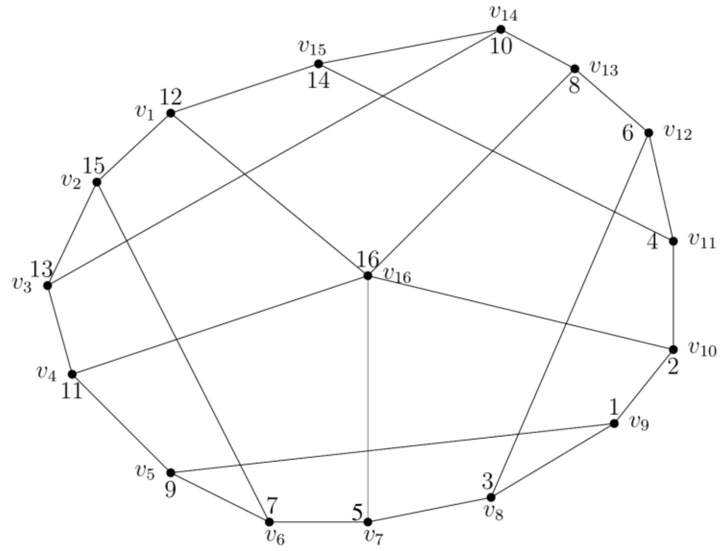

Figure 5. The prime cordial labeling of the hypohamiltonian graph.

We observe that $e_f(0)=12$, $e_f(1)=13$, so $|e_f(0)-e_f(1)|=1$. Hence the hypohamiltonian graph G is a prime cordial graph. □

Theorem 3.6. The cubic graph with 12 vertices is a prime cordial graph.

Proof.

Let G be the cubic graph with 12 vertices. Let $V(G)=\{v_1,\ldots,v_{12}\}$ and $E(G)=\{v_iv_{i+1} : 1\le i\le 5\} \cup \{v_iv_{i+1} : 7\le i\le 11\} \cup \{v_1v_6, v_7v_{12}, v_1v_7, v_2v_8, v_3v_5, v_4v_{10}, v_5v_9, v_6v_{11}\}$. Then $|V(G)|=12$ and $|E(G)|=18$.

$f(v_i)=2i$, $1\le i\le 6$; $f(v_7)=1$, $f(v_8)=11$, $f(v_9)=9$, $f(v_{10})=7$, $f(v_{11})=5$, $f(v_{12})=3$.

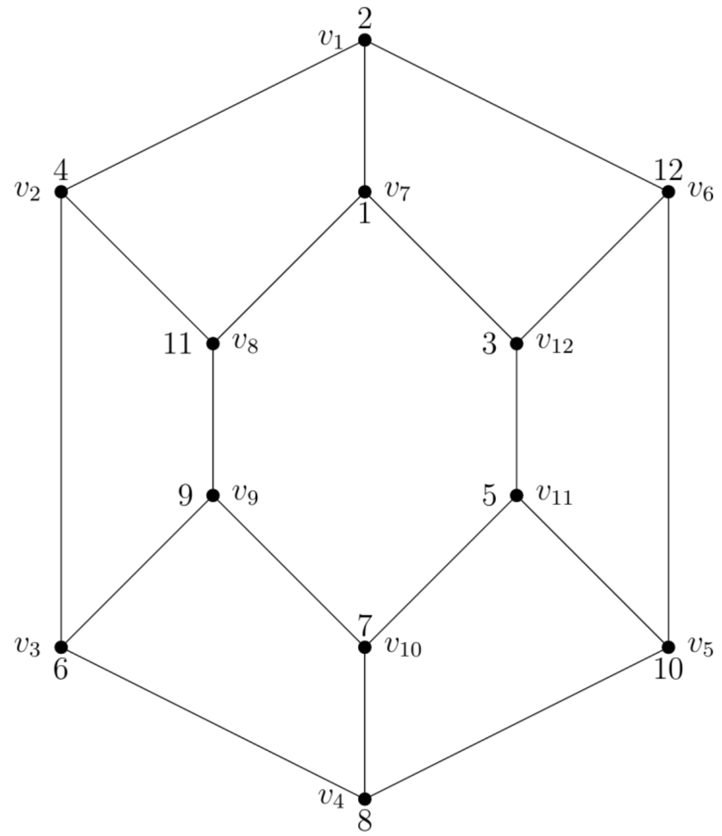

Figure 6. The prime cordial labeling of the cubic graph with 12 vertices.

We observe that e_f(0)=e_f(1)=9, so |e_f(0)−e_f(1)|=0. Hence the cubic graph with 12 vertices admits prime cordial labeling. □

Theorem 3.7. The crown graph C_n^+ is a prime cordial graph.

Proof.

Let C_n^+ be the crown graph with 2n vertices. Let $V(C_n^+)=\{v_1,\ldots,v_n,v_1',\ldots,v_n'\}$ and $E(C_n^+)=\{v_iv_i' : 1\le i\le n\} \cup \{v_iv_{i+1} : 1\le i\le n-1\} \cup \{v_1v_n\}$. Then $|V(C_n^+)|=2n$ and $|E(C_n^+)|=2n$.

$f(v_i)=2i,\ 1\le i\le n;\ \ f(v_i')=2i-1,\ 1\le i\le n.$

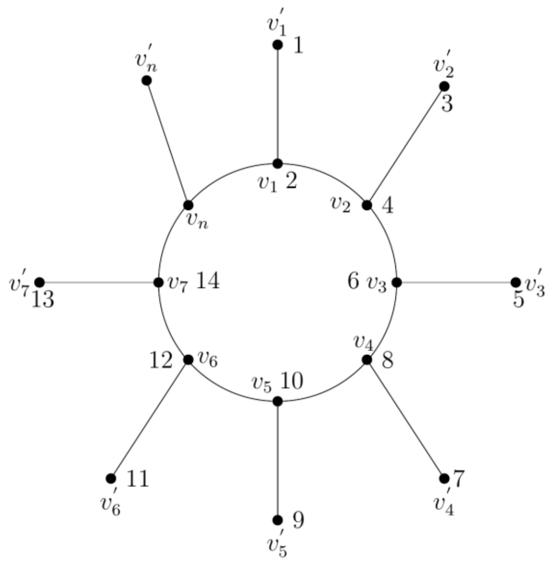

Figure 7. The prime cordial labeling of the crown graph C_n^+.

We observe that e_f(0)=e_f(1)=n, so |e_f(0)−e_f(1)|=0. Hence the crown graph C_n^+ is a prime cordial graph. □

4. Some New Results: Non-Prime-Cordial Graphs

In this section, we prove that the Herschel graph H_s, truncated tetrahedron graph, Moser spindle graph, and Wagner graph are not prime cordial graphs.

Theorem 4.1. The Herschel graph H_s is not a prime cordial graph.

Proof.

Let $\{v_1,v_2,\ldots,v_{11}\}$ be the vertices of the Herschel graph H_s with central vertex v_{10} and $E(H\_s)=\{v_iv_{i+1} : 1\le i\le 7\} \cup \{v_{10}v_{2i+1} : 0\le i\le 3\} \cup \{v_1v_8, v_9v_2, v_9v_4, v_9v_8, v_1v_4, v_{11}v_6, v_{11}v_8\}$. Then |V(H_s)|=11 and |E(H_s)|=18.

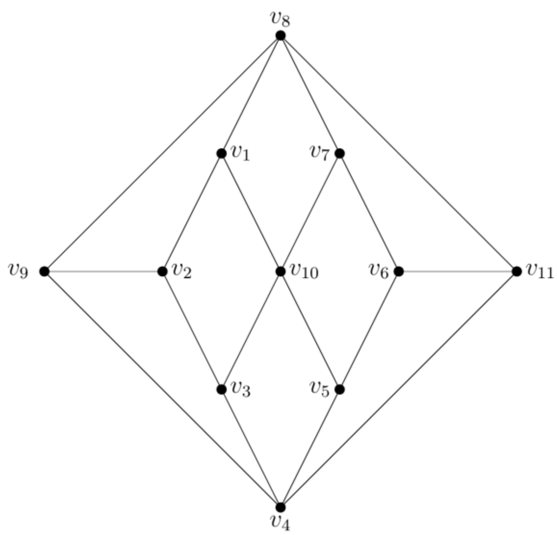

Figure 8. The Herschel graph H_s.

In graph H_s, only the pairs (2,4),(2,6),(2,8),(2,10),(3,6),(3,9),(4,6),(4,8),(5,10),(6,8),(6,10),(8,10) yield edge label 0. Any labeling contains at most eight such pairs, so e_f(0)≤8 and e_f(1)≥10. Thus |e_f(0)−e_f(1)|≥2. Hence H_s is not a prime cordial graph. □

Theorem 4.2. The Moser spindle graph is not a prime cordial graph.

Proof.

Let G be the Moser spindle graph. Let $V(G)=\{v_1,\dots,v_7\}$ and $E(G)=\{v_iv_{i+1} : 1\le i\le 4\} \cup \{v_1v_5, v_1v_6, v_2v_6, v_5v_6, v_3v_7, v_4v_7, v_5v_7\}$. Then $|V(G)|=7$ and $|E(G)|=11$.

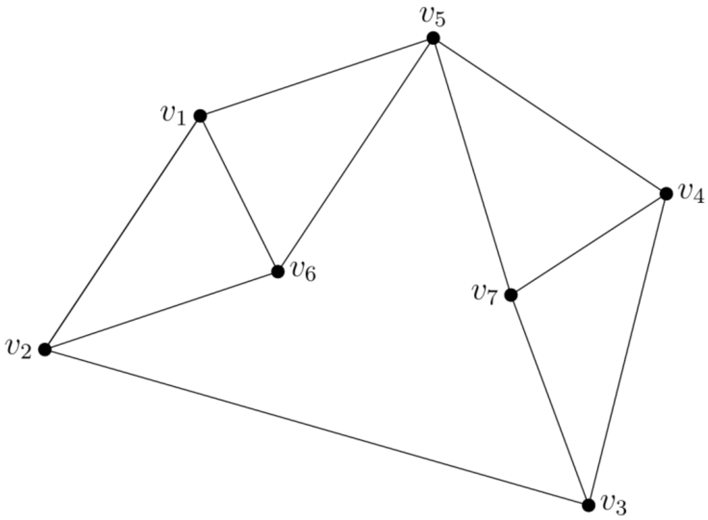

Figure 9. The Moser spindle graph.

Only pairs (2,4),(2,6),(3,6),(4,6) yield edge label 0. Hence $e\_f(0)\le 4$ and $e\_f(1)\ge 7$, giving $|e\_f(0)-e\_f(1)|\ge 3$. Hence the Moser spindle graph is not a prime cordial graph. □

Theorem 4.3. The Wagner graph is not a prime cordial graph.

Proof.

Let G be the Wagner graph. Let $V(G)=\{v_1,\dots,v_8\}$ and $E(G)=\{v_iv_{i+1} : 1\le i\le 7\} \cup \{v_1v_8, v_1v_5, v_2v_6, v_3v_7, v_4v_8\}$. Then $|V(G)|=8$ and $|E(G)|=12$.

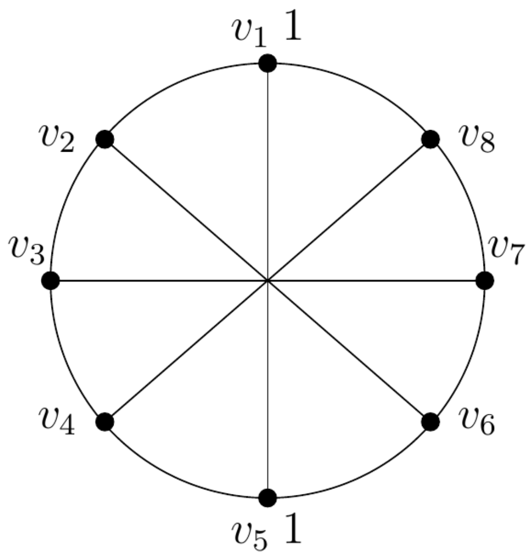

Figure 10. The Wagner graph.

Only pairs (2,4),(2,6),(2,8),(3,6),(4,6),(4,8),(6,8) yield edge label 0. Hence $e\_f(0)\le 4$ and $e\_f(1)\ge 8$, so $|e\_f(0)-e\_f(1)|\ge 4$. Hence the Wagner graph is not a prime cordial graph. □

Theorem 4.4. The truncated tetrahedron graph is not a prime cordial graph.

Proof.

Let G be the truncated tetrahedron graph. Let $V(G)=\{v_1,\dots,v_{12}\}$ and $E(G)=\{v_iv_{i+1} : 1\le i\le 7\} \cup \{v_1v_8, v_9v_1, v_9v_8, v_9v_{11}, v_{10}v_2, v_{10}v_3, v_{10}v_{12}, v_{11}v_4, v_{11}v_5, v_{12}v_6, v_{12}v_7\}$. The graph G has 12 vertices and 18 edges.

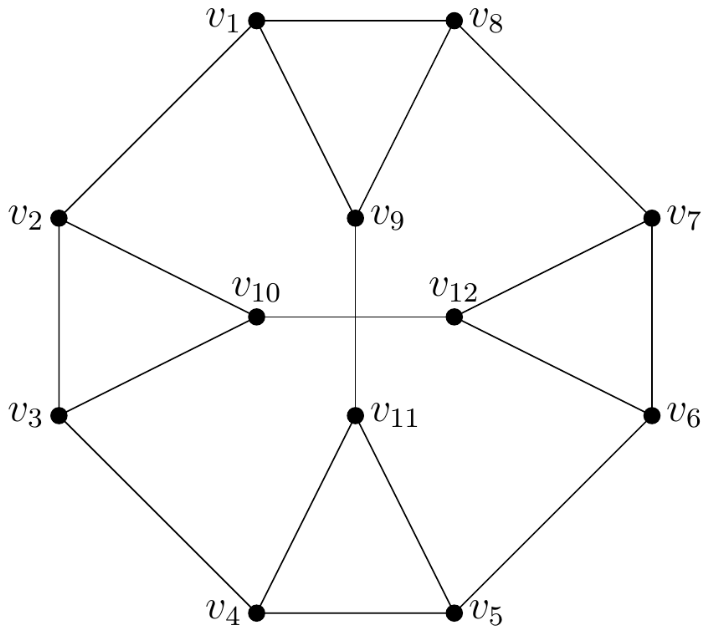

Figure 11. The truncated tetrahedron graph.

Only the pairs (2,4),(2,6),(2,8),(2,10),(2,12),(3,6),(3,9),(3,12),(4,6),(4,8),(4,12),(5,10),(5,12),(6,8),(6,10),(6,12),(8,10),(8,12),(10,12) yield edge label 0. Any labeling contains at most eight such pairs, so $e\_f(0)\leq 8$ and $e\_f(1)\geq 10$. Thus $|e\_f(0)-e\_f(1)|\geq 2$. Hence the truncated tetrahedron graph is not a prime cordial graph. □

Application: Prime Cordial Labeling in Epidemic Spread Networks

The prime cordial labeling framework introduced in the preceding sections — with its GCD-based edge-assignment rule (Definition 2.7) and near-balanced partition of edges into classes 0 and 1 — has direct applications in the mathematical modeling of epidemic disease spread. In this section we develop a three-part application that maps the results of Theorems 3.1–3.7 and Theorems 4.1–4.4 onto an epidemic contact network framework.

Contact Network Model and Prime Cordial Interpretation

In mathematical epidemiology, a contact network is a graph $G=(V,E)$ in which each vertex represents an individual or sub-population and each edge represents a potential disease transmission pathway. A prime cordial labeling $f:V(G)\rightarrow\{1,2,\ldots,p\}$ assigns each individual a unique immune status index. The induced edge label classifies every contact pathway: $f^*(uv)=1$ if $\gcd(f(u),f(v))=1$ (active transmission), and $f^*(uv)=0$ if $\gcd(f(u),f(v))>1$ (inactive/blocked).

The prime cordial condition $|e\_f(0)-e\_f(1)|\leq 1$ expresses a near-perfect balance between active and inactive transmission pathways, which has a direct implication for the basic reproduction number R_0. Graphs that do not admit prime cordial labeling (Theorems 4.1–4.4) model contact networks where no immune status assignment can achieve near-equal active and inactive pathways — the Herschel ($|e\_f(0)-e\_f(1)|\geq 2$), Moser spindle (≥ 3), Wagner (≥ 4), and truncated tetrahedron (≥ 2) represent structurally resistant epidemic networks.

SIR Dynamics on Prime Cordial Networks

We embed the standard SIR model onto the contact network G. The transmission rate along edge uv is weighted by $\beta\_uv=\beta_0\cdot f^*(uv)$. The effective reproduction number for vertex v is $R\_v=(\beta_0/\gamma)\cdot \deg_1(v)$, where $\deg_1(v)$ counts active (label-1) pathway neighbors. The network basic reproduction number is $R_0=(\beta_0/\gamma)\cdot \max\_v \deg_1(v)$. For the crown graph C_n^+, $R_0(C_n^+)=\beta_0/\gamma$, independent of n, making it a canonical model for hub-and-spoke epidemic containment.

Epidemic Data Encoding via Prime Cordial Labeling

The prime cordial labeling framework provides a mechanism for secure encoding of epidemic surveillance data. Since the prime cordial condition ensures $|e\_f(0)-e\_f(1)|\leq 1$, the encoded edge-label string has nearly equal numbers of 0s and 1s, providing maximum entropy and resistance to frequency-analysis attacks. Non-prime-cordial graphs cannot serve as codebooks since any encoding attempt is detectable via imbalance analysis.

Table 1. Prime cordial epidemic parameters for graphs in Theorems 3.1–3.7.

Graph	p	q	e_f(0)	e_f(1)	|e_f(0)−e_f(1)|	Epidemic Status
Durer (Thm 3.1)	12	18	9	9	0	Balanced; controllable
Heawood (Thm 3.2)	14	21	10	11	1	Near-balanced; controllable
Frucht (Thm 3.3)	12	18	9	9	0	Balanced; controllable
Tietze (Thm 3.4)	12	18	9	9	0	Balanced; controllable
Hypohamiltonian (Thm 3.5)	16	25	12	13	1	Near-balanced; controllable
Cubic-12 (Thm 3.6)	12	18	9	9	0	Balanced; controllable
Crown C_n^+ (Thm 3.7)	2n	2n	n	n	0	Perfectly balanced
Herschel (Thm 4.1)	11	18	≤8	≥10	≥2	Structurally resistant
Moser spindle (Thm 4.2)	7	11	≤4	≥7	≥3	Highly resistant
Wagner (Thm 4.3)	8	12	≤4	≥8	≥4	Most resistant
Truncated tetrahedron (Thm 4.4)	12	18	≤8	≥10	≥2	Structurally resistant

Table 2. Summary: Prime cordial epidemic network and encoding properties.

Graph	Theorem	Epidemic Network Property	Encoding Suitability
Durer	Thm 3.1	Perfectly balanced; R_0 halved	Secure codebook
Heawood	Thm 3.2	Near-balanced; controllable	Secure codebook
Frucht	Thm 3.3	Perfectly balanced	Secure codebook
Tietze	Thm 3.4	Perfectly balanced	Secure codebook
Hypohamiltonian	Thm 3.5	Near-balanced; scalable	Secure codebook
Cubic-12	Thm 3.6	Perfectly balanced	Secure codebook
Crown C_n^+	Thm 3.7	Hub-spoke; scale-free R_0	Canonical scalable model
Herschel H_s	Thm 4.1	Structurally resistant	Unsuitable (imbalanced)
Moser spindle	Thm 4.2	Highly resistant	Unsuitable (imbalanced)
Wagner	Thm 4.3	Most resistant; Δ≥4	Unsuitable (imbalanced)
Truncated tetrahedron	Thm 4.4	Structurally resistant	Unsuitable (imbalanced)

Concluding Remarks

In the present work we investigated prime cordial labeling for seven families of well-known graphs and proved four non-admissibility results. The graphs Durer, Heawood, Frucht, Tietze, hypohamiltonian, cubic (12 vertices), and crown C_n^+ are prime cordial (Theorems 3.1–3.7), while the Herschel, Moser spindle, Wagner, and truncated tetrahedron graphs are not (Theorems 4.1–4.4). Furthermore, as demonstrated in Section 5, the prime cordial labeling framework provides a rigorous mathematical basis for epidemic network analysis, hub-and-spoke epidemic modeling with scale-free reproduction number, and secure epidemic surveillance data encoding.

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