Linear Double Diffusive Convection In a Micropolar Liquid Occupying a Rectangular Box

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Abstract
This paper is a Fourier-series assisted analytical study of two-dimensional steady finite amplitude double diffusive convection in micropolar liquids occupying a rectangular box. The horizontal walls of the cavity are uniformly heated and salted to establish linear temperature and concentration distributions in the vertical direction. The vertical walls are insulated and impermeable. The expression for the critical Rayleigh number is obtained in closed form as a function of aspect ratio, solutal Rayleigh number, micropolar liquids parameters and diffusivity ratio. The results of slender vertical, square and shallow boxes are obtained as limiting cases of the present study.

Keywords: Double diffusive convection; Rayleigh number; Solutal Rayleigh number; Aspect ratio; Rectangular box.

1. Introduction

Natural convection in enclosures is and has long been a subject of intense research. It occurs in everyday life, for instance in double-glazed windows and car batteries, as well as in industrial processes such as crystal growth and electrochemical metal refining. The density differences which, when acted upon by gravity, cause convection are generally due to differences in either temperature or chemical composition (or both). In the following very limited review of the literature, we will focus on rectangular box where the flow is driven by either prescribed heat/mass fluxes or prescribed temperatures/concentrations at two opposing sides. Unless otherwise stated, two-dimensional geometries are considered. The double diffusive convection with microploar liquids has been studied by [7], [8] and [3] examined double diffusive convection using micropolar ferromagnetic fluid in the presence of magnetic field. They applied linear theory and normal mode methods to study the onset of convection. The roles of various micropolar parameters are shown graphically. [2] studied two component convection in a non-Newtonian fluid with permeable surface boundary in the presence of porous medium. They solved numerically the system of non-linear differential equations by using finite difference technique. They investigated the various parameters and compared the results with the existing results. [6] studied the thermosolutal convection of micropolar fluids in porous medium in the presence of rotation. They found the stability effect of rotation in the presence of salinity. Thus, the study of thermosolutal convection in porous medium in a fluid is of great importance. [4] investigated the double diffusive convection in a non-Newtonian fluid with porous medium. He considered the fluid in a truncated cone with constant temperature and concentration at the walls and transformed the complex surface of the cone into a smooth surface and hence solved the resulting differential equation by using cubic spline method. Finally, studied the various parameters of the fluid and also showed their effects graphically. [1] studied the role of heat and mass transfer in a micropolar fluid between two parallel plates with porous medium. They used homotropy analytical techniques to solve the differential equations. For different values of micropolar parameters, they found microrotation and velocity profiles. However, the study on double diffusive convection in micropolar liquid occupying a rectangular box is not available to the authors knowledge. Therefore, in the present paper we investigate the effects of couple stress and coupling parameter on the onset of double diffusive convection in a micropolar liquid.

Nomenclature

A Aspect ratio
b Width of rectangular box
C Couple stress parameter
$\vec{g}$ Acceleration due to gravity $(0, 0, -g)$
h Height of rectangular box
I Momentum of Inertia
N_1 Coupling parameter
N_3 Couple stress parameter
p Pressure
Pr Prandtl number
$\vec{q}$ Velocity vector (u, v, w)
Ra Rayleigh number
Ra_f Scaled finite-amplitude Rayleigh number
R_s Solutal Rayleigh number
Rs_f Scaled finite-amplitude Solutal Rayleigh number
s Deviation from static concentration

Sh Solute concentration
S_0 Solute concentration at the upper boundary
t time
T Liquid temperature
T_0 Temperature at the upper boundary
u Horizontal velocity component
w Vertical velocity component
x Horizontal cartesian coordinate
z Vertical cartesian coordinate

Greek symbols
β_S Co-efficient of solute expansion
β_T Co-efficient of thermal expansion
χ Thermal diffusivity
χ_S Solutal diffusivity
ΔS Concentration difference between the two horizontal plates
ΔT Temperature difference between the two horizontal plates
η Shear kinematic viscosity or Vortex viscosity
η' Spin viscosity coefficient
λ' Co-efficient of bulk viscosity
λ_2 strain retardation coefficient
ω $(0, \omega_y, 0)$ Spin vector
ψ Stream function
ρ Actual density
ρ_0 Reference density
θ Deviation from static temperature
∇ Vector Differential operator $\left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$
∇^2 Two-dimensional Laplacian operator $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)$
∇_A^2 Two-dimensional modified Laplacian operator $\left(A^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)$
τ Diffusivity ratio $\frac{\chi_s}{\chi}$
ζ Co-efficient of coupling viscosity or vortex viscosity

Subscripts
f Finite-amplitude quantity

Superscripts
$*$ Dimensionless quantity

2.Mathematical formulation

We consider two-dimensional double diffusive convection in micropolar liquids occupying a rectangular box with height h and breadth b. We choose a cartesian coordinate system with the $z-$axis in the vertical direction and x-axis in the horizontal direction. The horizontal walls are at $z = 0$ and $z = h$ and vertical walls at $x = 0$ and $x = b$ as shown in Fig. (1). The box is of infinite horizontal extent in the y-direction. The liquid is heated from below and cooled from above as in a typical Rayleigh-Bénard problem, the temperature difference between the bounding walls being ΔT .

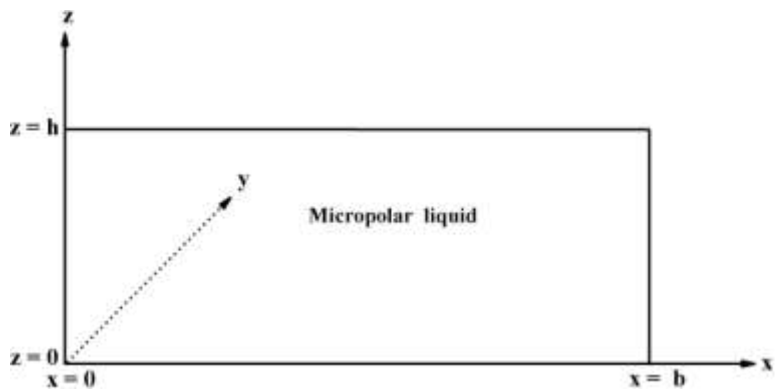

Figure 1: Schematic diagram of the flow configuration.

The governing equations are:

$$\nabla \cdot \vec{q} = 0 \tag{2.1}$$

$$\rho_0\left[\frac{\partial\vec{q}}{\partial t}+(\vec{q}\cdot\nabla)\vec{q}\right]=-\nabla p+\rho\vec{g}+(2\zeta+\eta)\nabla^2\vec{q}+\zeta(\nabla\times\vec{\omega}) \quad (2.2)$$

$$\rho_0 I\left[\frac{\partial\vec{\omega}}{\partial t}+(\vec{q}\cdot\nabla)\vec{\omega}\right]=(\lambda'+\eta')\nabla(\nabla\cdot\vec{\omega})+\eta'\nabla^2\vec{\omega}+\zeta(\nabla\times\vec{q})-2\zeta\vec{\omega} \quad (2.3)$$

$$\frac{\partial T}{\partial t}+(\vec{q}\cdot\nabla)T=\chi\nabla^2 T \quad (2.4)$$

$$\frac{\partial S}{\partial t}+(\vec{q}\cdot\nabla)S=\chi_s\nabla^2 S \quad (2.5)$$

$$\rho=\rho_0[1-\beta_T(T-T_0)+\beta_S(S-S_0)] \quad (2.6)$$

In the micro polar liquid the particle spin matches with the vorticity of the carrier liquid, and this results in an additional biharmonic term in Eq. (2.2). This is a drag term contributed to by the suspended particles. The effect of suspended particles in Eq. (2.4) on temperature T comes through the velocity $\vec{q}$. The upper and lower boundaries are at isothermal temperatures T_0 and $T_0+\Delta T_0$, and concentration S_0 and $S_0+\Delta S$, where ΔT and ΔS is positive temperature and concentration differences respectively. All the boundaries are assumed to be impermeable and perfectly heat conducting. From the governing Eq. (2.1) - (2.6) it follows that a motionless conduction state exists only if the static temperature distribution is independent of x and depends linearly on z. The present study is restricted to this case.
It is convenient to express the temperature and solute in the form

$$T=T_0+\Delta T\left(1-\frac{z}{h}\right)+\theta \quad (2.7)$$

where, θ is the deviation from the static temperature.

$$S=S_0+\Delta S\left(1-\frac{z}{h}\right)+s \quad (2.8)$$

where, s is the deviation from the static concentration.
The component form of Eq. (2.1) - (2.6) for steady flow are:
Since the flow is two−dimensional, we introduce the stream function ψ by

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \quad (2.9)$$

$$\frac{1}{\rho_0}\frac{\partial p}{\partial x}-\frac{(2\zeta+\eta)}{\rho_0}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right)+\frac{\zeta}{\rho_0}\left(\frac{\partial\omega_y}{\partial z}\right)+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=0 \quad (2.10)$$

$$\frac{1}{\rho_0}\frac{\partial p}{\partial z}-\frac{(2\zeta+\eta)}{\rho_0}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right)-\frac{\zeta}{\rho_0}\left(\frac{\partial\omega_y}{\partial x}\right)+\frac{\rho}{\rho_0}g+u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z}=0 \quad (2.11)$$

$$\eta'\left(\frac{\partial^2\omega_y}{\partial x^2}+\frac{\partial^2\omega_y}{\partial z^2}\right)+\zeta\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)-2\zeta\omega_y-\rho_0 I\left(u\frac{\partial\omega_y}{\partial x}+w\frac{\partial\omega_y}{\partial z}\right)=0 \quad (2.12)$$

$$\chi\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial z^2}\right)-u\frac{\partial T}{\partial x}-w\frac{\partial T}{\partial z}=0 \quad (2.13)$$

$$\chi_s\left(\frac{\partial^2 S}{\partial x^2}+\frac{\partial^2 S}{\partial z^2}\right)-u\frac{\partial S}{\partial x}-w\frac{\partial S}{\partial z}=0 \quad (2.14)$$

$$\text{where, } \rho=\rho_0\left[1-\beta_T\Delta T\left(1-\frac{z}{h}\right)-\beta_T\theta+\beta_s\Delta S\left(1-\frac{z}{h}\right)+\beta_S s\right]. \quad (2.15)$$

Since the flow is two-dimensional, we introduce the stream function ψ by

$$u=\frac{\partial\psi}{\partial z}\,,\ w=-\frac{\partial\psi}{\partial x} \quad (2.16)$$

Also define non-dimensional variables denoted by asterisks

$$(x^*,z^*)=\left(\frac{x}{b},\frac{z}{h}\right),T^*=\frac{T}{\Delta T},\ S^*=\frac{S}{\Delta S},\ \omega_y^*=\frac{\omega_y}{\left(\frac{\chi}{h^2}\right)},\theta^*=\frac{\theta}{\Delta T},\psi^*=\frac{\psi}{\chi} \quad (2.17)$$

By eliminating pressure between equations. (2.10) and (2.11), introducing the stream function as given in (2.16) and introducing equation (2.17) into the resulting equation and in equations (2.12) – (2.14) and dropping the asterisk, we get the governing equations in the following form:

$$(1+N_1)\nabla_A^4\psi-N_1\nabla_A^4\omega_y-A^4 Ra\frac{\partial\theta}{\partial x}+A^4 Rs\frac{\partial S}{\partial x}=-\frac{A}{P_r}\frac{\partial(\psi,\nabla_A^2\psi)}{\partial(x,z)} \quad (2.18)$$

$$N_3\nabla_A^2\omega_y + N_1\nabla_A^4\psi - 2N_1\omega_y = -A\frac{N_2}{P_r}\frac{\partial(\psi,\omega_y)}{\partial(x,z)} \tag{2.19}$$

$$\nabla_A^2\theta - A\frac{\partial\psi}{\partial x} = -A\frac{\partial(\psi,\theta)}{\partial(x,z)} \tag{2.20}$$

$$\tau\nabla_A^2 s - A\frac{\partial\psi}{\partial x} = -A\frac{\partial(\psi,s)}{\partial(x,z)} \tag{2.21}$$

where all the parameters have been defined in nomenclature of this paper and $\frac{\partial(\ ,\ )}{\partial(x,\ z)}$ is the Jacobian.
In writing equations (2.18) – (2.21) use has been made of equations (2.6). Equations (2.18) – (2.21) are solved for stress free, isothermal, isohaline and vanishing spin boundary conditions used in the study are:

$$\psi = \nabla_A^2\psi = \omega_y = \theta = s = 0 \quad \text{at} \quad z = 0,\ 1 \ \text{ for } 0 < x < 1, \tag{2.22}$$
$$\psi = \nabla_A^2\psi = \omega_y = \frac{\partial\theta}{\partial x} = \frac{\partial s}{\partial x} = 0 \quad \text{at} \quad x = 0,\ 1 \ \text{ for } 0 < z < 1,$$

were, $\nabla_A^2 = A^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$.

3. Linear Stability theory for stationary convection

The onset of double diffusive convection in micropolar liquid by the linear versions of equations (2.18) – (2.21). To make this study we neglect the Jacobians in equations (2.18) – (2.21) to get governing differential equations in the form:

$$(1 + N_1)\nabla_A^4\psi - N_1\nabla_A^4\omega_y - A^4R_a\frac{\partial\theta}{\partial x} + A^4R_s\frac{\partial S}{\partial x} = 0 \tag{3.1}$$
$$N_3\nabla_A^2\omega_y + N_1\nabla_A^4\psi - 2N_1\omega_y = 0 \tag{3.2}$$

$$\nabla_A^2\theta - A\frac{\partial\psi}{\partial x} = 0 \tag{3.3}$$

$$\tau\nabla_A^2 s - A\frac{\partial\psi}{\partial x} = 0 \tag{3.4}$$

Eliminating ω_y between equations (3.1) and (3.2) we get

$$(1 + N_1)N_3\nabla_A^6\psi - (2 + N_1)N_1\nabla_A^4\psi - A^4R_a\left[N_3\frac{\partial}{\partial x}(\nabla_A^2\theta) - 2N_1\frac{\partial\theta}{\partial x}\right] + \tag{3.5}$$
$$A^4R_s\left[N_3\frac{\partial}{\partial x}(\nabla_A^2 s) - 2N_1\frac{\partial s}{\partial x}\right] = 0$$

Taking solutions for the equations (3.3) -(3.5) in the form

$$\psi = \psi_0\sin(\pi x)\sin(\pi z), \tag{3.6}$$

$$\theta = \theta_0\cos(\pi x)\sin(\pi z), \tag{3.7}$$

$$s = s_0\cos(\pi x)\sin(\pi z), \tag{3.8}$$

Substtituting equations (3.6) -(3.8) in equations (3.3) -(3.5) we ge

$$[(1 + N_1)N_3L^6 + (2 + N_1)N_1L^4]\psi_0 + \pi A^4R_a[N_3L^2 + 2N_1]\theta_0 - \pi A^4R_s[N_3L^2 + 2N_1]s_0 = 0 \tag{3.9}$$

$$\pi A\psi_0 + L^2\theta_0 = 0 \tag{3.10}$$

$$\pi A\psi_0 + \tau L^2 s_0 = 0 \tag{3.11}$$

where $L^2 = \pi^2(A^2 + 1)$.

For non-trivial solution for ψ_0, θ_0 & s_0 we require

$$\begin{vmatrix} (1 + N_1)N_3L^6 + (2 + N_1)N_1L^4 & \pi A^4R_a[N_3L^2 + 2N_1] & -\pi A^4R_s[N_3L^2 + 2N_1] \\ \pi A & L^2 & 0 \\ \pi A & 0 & \tau L^2 \end{vmatrix} = 0 \tag{3.12}$$

Solving equation (3.12) for R_a we obtain

$$R_a = \frac{L^6[(1+N_1)N_3L^2+(2+N_1)N_1]}{\pi^2A^5[N_3L^2+2N_1]} + \frac{R_s}{\tau} \tag{3.13}$$

In the absence of solute concentration $R_s = 0$, equation (3.13) reduces to

$$R_a = \frac{L^6[(1+N_1)N_3L^2+(2+N_1)N_1]}{\pi^2A^5[N_3L^2+2N_1]} \tag{3.14}$$

Setting $N_1 = 0$, and keeping N_3 arbitrary in equation (3.14), we get

$$R_a = \frac{\pi^4(A^2+1)^3}{A^5} \tag{3.15}$$

4. Results and discussion

The onset of double diffusive convection in a Micropolar liquids occupying a rectangular box is discussed using linear theory. In the present paper, the behavior of the system as a function of the aspect ratio A , solute Rayleigh number R_s, coupling parameter N_1 , couple stress parameter N_3 and the diffusivity ratio τ. Different values of A are taken to study convection in rectangular, square and slender vertical slots. Figs. (2) - (5) show the variation of R_a and R_{af} with A for different values of $R_s, N_1,$ N_3 & τ. We observe from the figures that both Ra and Raf decrease with increase in A. Small values of A (< 1) correspond to a shallow box and large values of A correspond to a vertical slender slot. The value $A = 1$ refers to a square box. In comparison with a square box, in the case of a shallow box more energy is required to drive the convective flow. In the case of a slender slot, however, less energy is required to drive the flow. The above facts are demonstrated in Figs. (2) - (5). Fig. (3) depicts that the of R_a and R_{af} increase with increase in N_1, indicating that the effect of coupling parameter is to stabilize the system. Fig. (4) is the variation of R_a and R_{af} with couple stress parameter N_3 and we have observed that R_a and R_{af} increase with increase in N_3 and decrease with increase in A. Fig. (5) depicts that the R_a decreases with increase in diffusivity ratio and R_{af} increases with increase in τ . The effect of τ on R_a and R_{af} is in opposed way which we unable to suitably explain at the present time. The sensitivity of the Rayleigh number R_a and $R_{af}$for steady double diffusive convection in micropolar liquids in rectangular, square and slender vertical slot is shown in Figs. (6) - (9). Fig. (6) depicts that the R_a and R_{af}increase with increase in R_s and decrease with increase in A, which shows that R_a and $R_{af}$is more in the case of rectangular box than square and slender vertical slot. The variation of Ra and Raf versus N_1 is shown in Fig. (7). From this we have noticed that R_a and R_{af} increase with increase with N_1 and decrease with increase with A. Fig. (8) depicts that the R_a and R_{af}increase with increase in N_3 and decreases with A. Fig. (9) gives the variation of R_a and R_{af}versus τ is shown and from this we have noticed that R_a decreases with increase in τ and decreases with increase in A, while R_{af} increases with increase in τ and decreases with increase in A. As a general result we note that sub-critical motion is possible for all combinations of parameters [see Figs. 2 - (9)].

5. Conclusion

The following conclusion can be drawn from the study on Linear and Non-Linear Double Diffusive Convection in a Micropolar Liquid Occupying a Rectangular Box:
The effect of Rs, N1 and N3 is to stabilize the system in double diffusive convection in a micropolar liquid. The effect of τ is to destabilize the system in double diffusive convection in a micropolar liquid.

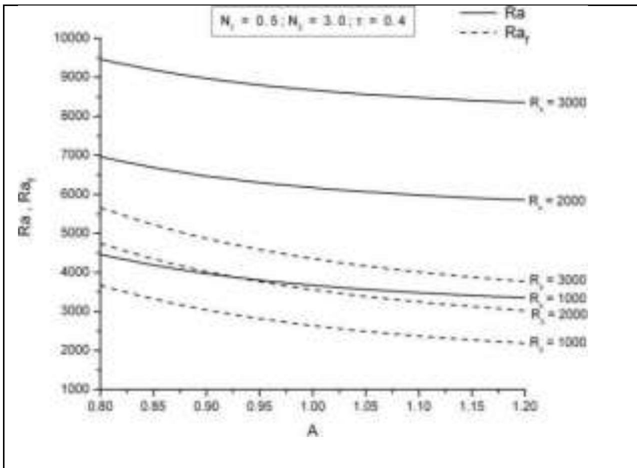

Figure 2: Plot of Ra and Raf versus A for different values of Rs.

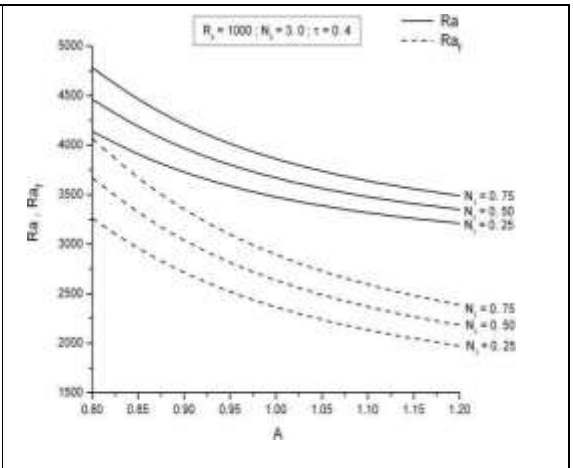

Figure 3: Plot of Ra and Raf versus A for different values of N1.

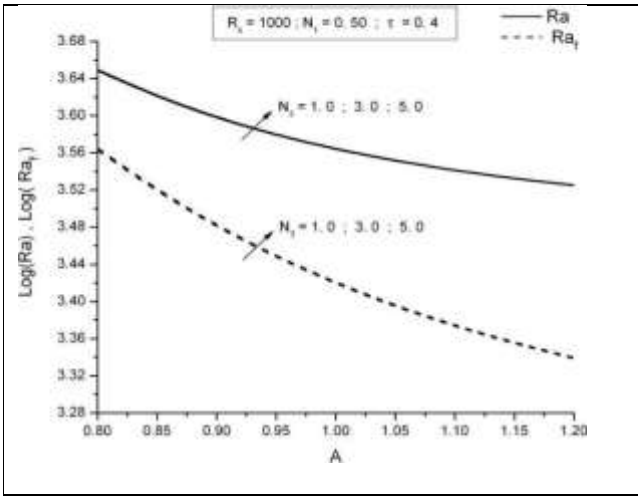

Figure 4: Plot of Ra and Raf versus A for different values of N3.

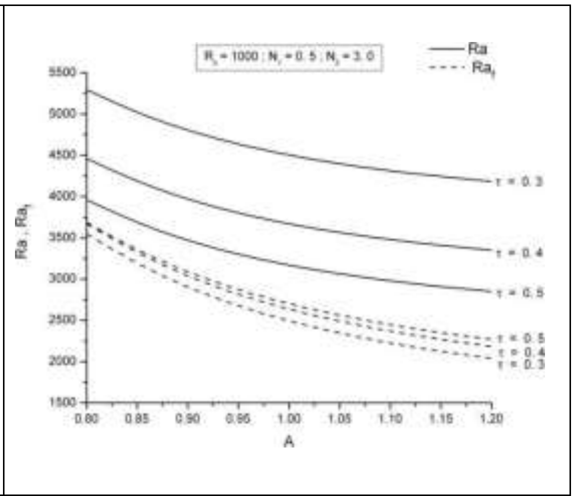

Figure 5: Plot of Ra and Raf versus A for different values of τ

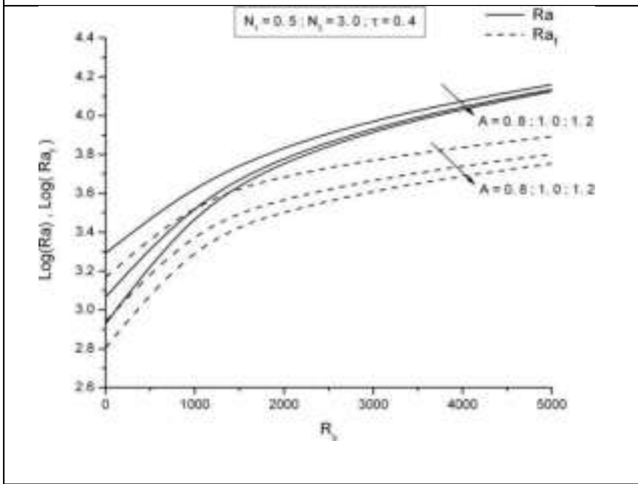

Figure 6: Plot of Ra and Raf versus Rs for different values of A.

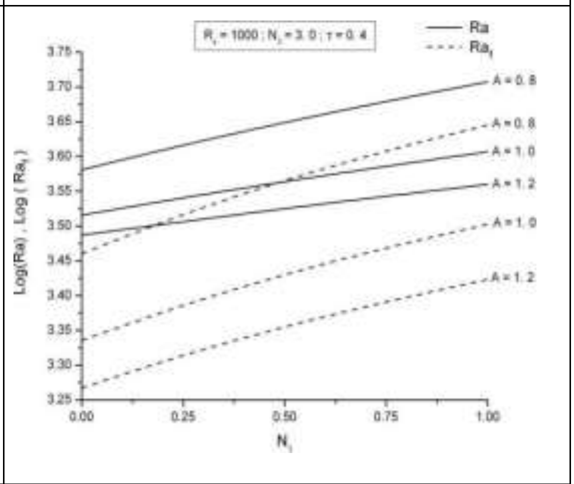

Figure 7: Plot of Ra and Raf versus N1 for different values of A.

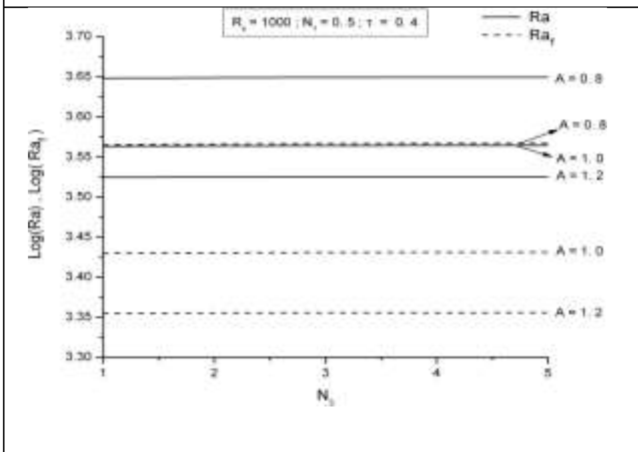

Figure 8: Plot of Ra and Raf versus N3 for different values of A.

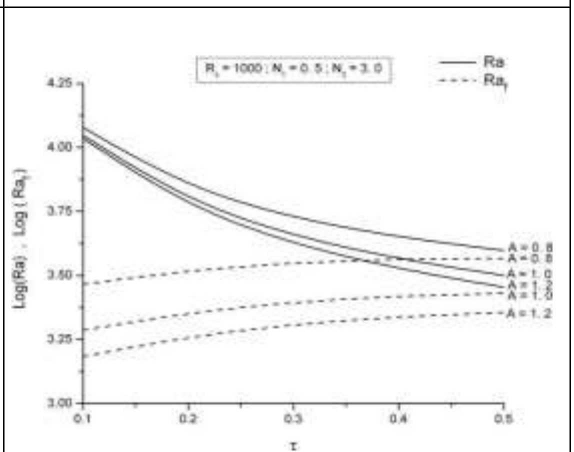

Figure 9: Plot of Ra and Raf versus τ for different values of A.

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