Score and Distance Measures in Q-Complex Fermatean Neutrosophic Set Theory

Vijaya V [1], Rajalaxmi D [2*]

[1,2*]PG & Research Department of Mathematics, Seethalakshmi Ramaswami College,(Autonomous), Tiruchirappalli, (Affiliated to Bharathidasan University, Tiruchirappalli), India.
Email: vvijayamanikandan@gmail.com[1], rajalaxmimat@gmail.com[2]

Abstract

Q-complex neutrosophic sets (Q-CNS) and Q-complex Fermatean neutrosophic sets (Q-CFNS) represent recent extensions of fuzzy set theory that introduce complex-valued membership degrees to better capture ambiguity. Q-CNS builds on neutrosophic sets by expressing the truth, indeterminacy, and falsity components as complex numbers, allowing for richer and more flexible representation of uncertainty. Q-CFNS advances this framework further by adopting Fermatean-type membership grades, which offer improved precision in capturing imprecise information. This added structure supports more nuanced applications across decision-making, artificial intelligence, and data analysis, allowing complex relationships and vague data to be modeled with greater fidelity. In essence, these are complex-valued sets governed by Fermatean-type constraints, designed to comprehensively handle multiple dimensions of uncertainty. Their strength becomes particularly evident in situations where conventional fuzzy sets fall short — for instance, when dealing with inconsistent or partially missing data. By leveraging Q-CNS and Q-CFNS, researchers and practitioners gain a more robust toolset for modeling complex real-world systems. In this article, we introduce and define score and distance functions specifically for Q-CFNS.

Keywords: Neutrosophic Sets, Complex Neutrosophic Sets, Compex Fermatean Neutrosophic Sets,Q-Neutrosophic Sets, Q-Complex Fermatean Neutrosophic Set

Introduction

The introduction of fuzzy sets by Zadeh [1] opened the way for addressing problems involving uncertainty. Building on this, Atanassov [2] incorporated a non-membership function into fuzzy sets to formulate intuitionistic fuzzy sets, which proved useful in tackling practical problems. Smarandache [3] subsequently proposed neutrosophic sets (NS) and neutrosophic logic as a broader generalization encompassing both fuzzy sets [1] and intuitionistic fuzzy sets [2].

Several researchers have since extended these ideas into Q-fuzzy sets (Q-FSs), intuitionistic Q-fuzzy sets (Q-IFS), and Q-neutrosophic sets (Q-NS). Jun [4] first introduced Q-fuzzy subalgebras within BCK/BCI-algebras, while Roh et al. [5] examined intuitionistic Q-fuzzy subalgebras in the same algebraic setting. Kazanci et al. [6] introduced intuitionistic Q-fuzzification of R-subgroups (subnear-rings) in near-rings and explored their associated properties.

Muthuraj et al. [7] proposed and analyzed the anti-Q-fuzzy group along with its lower-level subgroups. Nayagam et al. [8] formulated the intuitionistic Q-fuzzy HX group as a novel algebraic construct. Sithar Selvam et al. [9] provided a detailed treatment of the anti-Q-fuzzy R-closed KU-ideal in KU-algebras and its lower-level cuts. Tanamoon et al. [10] defined and characterized Q-fuzzy UP-ideals and Q-fuzzy UP-subalgebras within UP-algebras. To address two-dimensional data marked by ambiguity, inconsistency, and indeterminacy, Abu Qamar and Hassan [11] developed the Q-NS framework. Thiruveni and Solairaju [12], meanwhile, formulated neutrosophic Q-fuzzy sets, derived results on neutrosophic Q-fuzzy subgroups, and further extended this work to multi Q-neutrosophic sets.

The integration of soft set theory with Q-FS, Q-IFS, and Q-NS has also drawn considerable attention. Adam and Hassan [13] combined soft sets with Q-FS to introduce the Q-fuzzy soft set. Broumi [14] proposed the Q-intuitionistic fuzzy soft set (Q-IFSS), establishing several of its basic properties and operations. Mahmood et al. [15] put forward the Q-single valued neutrosophic soft set and the multi Q-single valued neutrosophic set, along with foundational results and related properties. Ashraf Al-Quran et al. [16] introduced the Q-complex neutrosophic set.

Separately, Ramot et al. [17] introduced complex fuzzy sets, a concept later extended into complex neutrosophic sets [18] and complex intuitionistic fuzzy sets [19]. Antony and Jansi [20] proposed Fermatean neutrosophic fuzzy sets, which Broumi et al. [21] subsequently extended into complex Fermatean neutrosophic sets. Vijaya et al. [22] defined set-theoretic operations for complex Fermatean neutrosophic sets and introduced a new distance measure for solving decision-making problems.

For addressing critical path problems, Thangaraj Beaula and Vijaya [23–25] applied various fuzzy numbers together with ranking techniques. Vijaya et al. [26–28] further employed Pythagorean and neutrosophic fuzzy numbers, as well as cubic bipolar complex neutrosophic sets, to solve critical path and decision-making problems.

The Q-complex Fermatean neutrosophic set [29] (Q-CFNS) is an advanced mathematical construct, rooted in neutrosophic and fuzzy set theory, designed to represent ambiguous, inconsistent, and uncertain information. Each element is characterized by three complex-valued membership degrees — truth, indeterminacy, and falsity —

which, beyond simply indicating membership strength, also capture phase or periodic information through their complex-number structure. The Fermatean constraint, requiring the powers of the truth and falsity memberships to satisfy a defined relationship, grants the model greater flexibility than earlier fuzzy-based approaches. As a result, Q-CFNS finds broad application in data analysis, artificial intelligence, and decision-making contexts characterized by uncertainty or imprecision. Some algebraic and operational properties were discussed by Vijaya et al[29]. In this paper we define the score function and distance measures of Q-complex Fermatean neutrosophic sets.

Preliminaries

Fuzzy Set[1]

Let X be a Universal Set. Then a Fuzzy Set $\widetilde{A}$ is usually characterized by the membership function $\mu_{\widetilde{A}}(x)$, where $\mu_{\widetilde{A}}(x): X \to [0,1]$ and it is denoted by $\widetilde{A} = \{x, \mu_{\widetilde{A}}(x); x \in X\}$

Q-Fuzzy Set[4]

Let X and Q be nonempty sets. A Q-Fuzzy set in X is defined by an arbitrary function $f : X \times Q \to [0,1]$.

Intutitionistic Q-Fuzzy Set[5]

Let X and Q be nonempty sets. An Intuitionistic Q-Fuzzy subset A in X is defined as follows
$A = \{\langle (x,q); \mu_A(x,q), \nu_A(x,q) \rangle; x \in X, q \in Q\}$, where $\mu_A : X \times Q \to [0,1]$ & $\nu_A : X \times Q \to [0,1]$ are the degree of membership and non membership of the element $(x,q) \in X \times Q$, $0 \le \mu_A(x) + \nu_A(x) \le 1$

Q-Neutrosophic Set[11]

Let X be an Universal set and Q be a nonempty set. A Q-Neutrosophic set A in X and Q is given by
$A = \{\langle (x,q); T_A(x,q), I_A(x,q), F_A(x,q) \rangle; x \in X, q \in Q\}$, where $T_A; I_A; F_A : X \times Q \to \left]{}^{-}0; 1^{+}\right[$ are the Truth Membership Function, Indeterminacy Membership function and Falsity Membership function respectively with ${}^{-}0 \le T_A + I_A + F_A \le 3^{+}$

Multi Q-Neutrosophic Set[12]

Let X be an Universal set and Q be a nonempty set. L is the unit interval [0.1] & n is a positive integer. A multi Q-Neutrosophic set A in X and Q is given by
$A = \{\langle (x,q); T_{A_i}(x,q), I_{A_i}(x,q), F_{A_i}(x,q) \rangle; x \in X, q \in Q\}$, $\forall i = 1,2,3,..n$ where $T_{A_i}; I_{A_i}; F_{A_i} : X \times Q \to L^n$ are the Truth Membership Function, Indeterminacy Membership function and Falsity Membership function respectively for each $x \in X, q \in Q$ with $0 \le T_{A_i} + I_{A_i} + F_{A_i} \le 3$, $\forall i = 1,2,3,..n$ and n is the dimension of A.

Complex Fuzzy Set[17]

Complex Fuzzy Set is an extension of Fuzzy set where the range is a unit circle in the complex plane. The membership function is complex valued which is given by $\mu_{\widetilde{A}}(x) = r(x)e^{i\theta(x)}$, where $r(x) \in [0,1], \theta(x) \in [0,2\pi]$

Complex Neutrosophic Set [18]

A Neutrosophic set can be extended to Complex Neutrosophic Set where the Truth membership, Indeterminacy membership and Falsity membership functions are all complex valued functions and is given by $A = \{x, T_A(x), I_A(x), F_A(x), x \in X\}$ where $T_A(x) = r_A(x)e^{i\theta_1(u)}$, $I_A(x) = s_A(x)e^{i\theta_2(u)}$, $F_A(x) = t_A(x)e^{i\theta_3(u)}$, $0^{-} \le r_A + s_A + t_A \le 3^{+}$, $0 \le \theta_1 + \theta_2 + \theta_3 \le 2\pi$

Fermatean Neutrosophic Set[20]

A Fermatean Neutrosophic Set A defined on X is represented as
$A = \{x, T_A(x), I_A(x), F_A(x), x \in X\}$
with the condition $0^{-} \le T_A^3 + I_A^3 + F_A^3 \le 2^{+}$ and $0 \le T_A^3 + F_A^3 \le 1$, $0 \le I_A^3 \le 1$

Complex Fermatean Neutrosophic Set[21]

A Complex Fermatean Neutrosophic Set A on X is denoted by
$A = \{x, T_A(x), I_A(x), F_A(x), x \in X\}$, where $T_A(x) = r_A(x)e^{2\pi i\theta_1(u)}$, $I_A(x) = s_A(x)e^{2\pi i\theta_2(u)}$, $F_A(x) = t_A(x)e^{2\pi i\theta_3(u)}$, represents the truth, indeterminacy and falsity membership functions, where $r_A, s_A, t_A, \theta_1, \theta_2, \theta_3 : X \times Q \to [0,1]$ with the condition $0^{-} \le r_A^3 + s_A^3 + t_A^3 \le 3^{+}$, $0^{-} \le \theta_1^3 + \theta_2^3 + \theta_3^3 \le 3^{+}$. The triplet $(re^{2\pi i\theta_1}, se^{2\pi i\theta_2}, te^{2\pi i\theta_3})$ is called a complex fermatean neutrosophic number.

Q-COMPLEX FERMATEAN NEUTROSOPHIC SET(Q-CFNS) [25]

Let X and Q be nonempty sets. A Q-Complex Fermatean Neutrosophic Set A on X is denoted by $A=\{(x,q),T_A(x,q),I_A(x,q),F_A(x,q),x\in X,q\in Q\}$, where $T_A(x,q)=r_A(x,q)e^{2\pi i\theta_1(x,q)}$, $I_A(x,q)=s_A(x,q)e^{2\pi i\theta_2(x,q)}$, $F_A(x,q)=t_A(x,q)e^{2\pi i\theta_3(x,q)}$, represents the truth, indeterminacy and falsity membership functions with the condition $0^-\le r_A^3(x,q)+s_A^3(x,q)+t_A^3(x,q)\le 3^+$, $0^-\le \theta_1^3(x,q)+\theta_2^3(x,q)+\theta_3^3(x,q)\le 3^+$.

If in the above definition we let $\theta_1(x,q),\theta_2(x,q),\theta_3(x,q)$ equal to zero then the Q-CFNS reduces to QFNS

Multi Q-Complex Fermatean Neutrosophic Set[25]

Let X and Q be nonempty sets. A multi Q-Complex Fermatean Neutrosophic Set A on X is denoted by $A=\{(x,q),T_{A_i}(x,q),I_{Ai}(x,q),F_{Ai}(x,q),x\in X,q\in Q\}$, where $T_{A_i}(x,q)=r_{A_i}(x,q)e^{2\pi i\theta_{1A_i}(x,q)}$, $I_{A_i}(x,q)=s_{Ai}(x,q)e^{2\pi i\theta_{2A_i}(x,q)}$, $F_{A_i}(x,q)=t_{A_i}(x,q)e^{2\pi i\theta_{3A_i}(x,q)}$, represents the truth, indeterminacy and falsity membership functions where $r_{A_i},s_{Ai},t_{A_i},\theta_{1A_i},\theta_{2A_i},\theta_{3A_i}:X\times Q\to L^n$ such that $0^-\le r_{A_1}^3(x,q)+s_{A_1}^3(x,q)+t_{A_1}^3(x,q)\le 3^+$, $0^-\le \theta_{1A_1}^3(x,q)+\theta_{2A_1}^3(x,q)+\theta_{3A_1}^3(x,q)\le 3^+$.

Remark:

i. If n=1, then the multi Q-Complex Fermatean Neutrosophic Set reduces to Q-Complex Fermatean Neutrosophic set.

ii. If n=1 and $\theta_{1A_i},\theta_{2A_i},\theta_{3A_i}=0$, then the multi Q-Complex Fermatean Neutrosophic Set reduces to Q-Fermatean Neutrosophic set.

iii. A QCFNS A is said to a zero QCFNS denoted by A_ϕ, if $T_A(x,q)=0$, $I_A(x,q)=1$, $F_A(x,q)=1$ $\forall x\in X,q\in Q$ and is denoted as $A_\phi=\{(x,q);0,1,1;x\in X,q\in Q\}$, with the condition $0^-\le 0^3+1^3+1^3\le 3^+$

iv. A QCFNS is said to be unit QCFNS, denoted by A_ψ, $T_A(x,q)=1$, $I_A(x,q)=0$, $F_A(x,q)=0$ $\forall x\in X,q\in Q$ and is denoted as $A_\psi=\{(x,q);1,0,0;x\in X,q\in Q\}$, with the condition $0^-\le 1^3+0^3+0^3\le 3^+$

Score Function:

Let $A=\{(x,q)r_1(x,q)e^{2\pi i\theta_1(x,q)},s_1(x,q)e^{2\pi i\theta_2(x,q)},t_1(x,q)e^{2\pi i\theta_3(x,q)},x\in X,q\in Q)\}$, be a Q-Complex Fermatean Neutrosphic Set. The Score Function of A is given by

$$S(A)=\frac{1}{6}\left(r_1(x,q+(1-s_1(x,q))+(1-t_1(x,q))+\theta_1(x,q)+(1-\theta_2(x,q)+(1-\theta_3(x,q))\right)$$

Hamming Distance Measure:

If $A=\{(x,q)r_1(x,q)e^{2\pi i\theta_1(x,q)},s_1(x,q)e^{2\pi i\theta_2(x,q)},t_1(x,q)e^{2\pi i\theta_3(x,q)},x\in X,q\in Q)\}$ &
$B=\{(x,q),r_2(x,q)e^{2\pi i\phi_1(x,q)},s_2(x,q)e^{2\pi i\phi_2(x,q)},t_2(x,q)e^{2\pi i\phi_3(x,q)},x\in X,q\in Q\}$ are two Q-CFNS, the Hamming Distance Measure between A & B is defined as

$$d_H(A,B)=\frac{1}{6n}\sum_{j=1}^{n}\left|r_1(x,q)-r_2(x,q)\right|+\left|s_1(x,q)-s_2(x,q)\right|+\left|t_1(x,q)-t_2(x,q)\right|$$
$$+\left|\theta_1(x,q)-\phi_1(x,q)\right|+\left|\theta_2(x,q)-\phi_2(x,q)\right|+\left|\theta_3(x,q)-\phi_3(x,q)\right|$$

Euclidean Distance Measure:

If $A = \{(x,q)r_1(x,q)e^{2\pi i\theta_1(x,q)}, s_1(x,q)e^{2\pi i\theta_2(x,q)}, t_1(x,q)e^{2\pi i\theta_3(x,q)}, x \in X, q \in Q)\}$ &

$B = \{(x,q), r_2(x,q)e^{2\pi i\phi_1(x,q)}, s_2(x,q)e^{2\pi i\phi_2(x,q)}, t_2(x,q)e^{2\pi i\phi_3(x,q)}, x \in X, q \in Q\}$ are two Q-CFNS, the Euclidean Distance Measure between A & B is defined as

$$d_H(A,B) = \frac{1}{6n}\sum_{j=1}^{n}(r_1(x,q)-r_2(x,q))^2 + (s_1(x,q)-s_2(x,q))^2 + (t_1(x,q)-t_2(x,q))^2$$

$$+(\theta_1(x,q)-\phi_1(x,q))^2 + (\theta_2(x,q)-\phi_2(x,q))^2 + (\theta_3(x,q)-\phi_3(x,q))^2$$

Score functions and distance measures for Q-Complex Fermatean Neutrosophic Sets (Q-CFNS) support a range of applications involving uncertain, periodic, or conflicting data. Score functions aid multi-criteria decision making by ranking alternatives in problems like supplier selection and risk assessment, while distance measures enable pattern recognition, medical diagnosis, and consensus modeling by quantifying similarity or disagreement between data points. The framework's ability to capture phase and periodic information also makes it well-suited to time-series analysis, critical path scheduling, and AI applications such as clustering and classification under two-dimensional uncertainty.

Limitations:

Despite its advantages, the proposed Q-CFNS framework has some limitations. The use of three complex-valued membership degrees increases computational complexity compared to simpler fuzzy models, and interpreting the phase components in real-world decision-making remains challenging. The proposed score and distance functions are not uniquely defined, so different formulations may yield varying rankings, raising concerns about consistency. Additionally, validation has largely relied on illustrative numerical examples rather than large-scale real-world datasets, and comparative benchmarking against other hybrid neutrosophic models remains limited.

Future Scope:

Future work could focus on developing more computationally efficient algorithms for large-scale applications and exploring alternative score and distance formulations to improve consistency. Validating the framework on real-world datasets in areas such as medical diagnosis and risk assessment, along with benchmarking against other hybrid neutrosophic models, would strengthen its practical relevance. The framework could also be extended with new aggregation operators and similarity measures, and integrated with machine learning for automated decision-making, as well as applied to emerging domains like IoT networks and time-series forecasting.

Conclusion

In this article, we have studied the concept of Q-Complex Fermatean Neutrosophic Sets (Q-CFNS), a mathematical framework that extends neutrosophic and Fermatean fuzzy set theory by incorporating complex-valued membership degrees to more effectively capture ambiguous, inconsistent, and periodic information. We proposed a score function to facilitate the ranking and comparison of Q-CFNS elements, along with Hamming and Euclidean distance measures to quantify the degree of dissimilarity between them. The proposed score function and distance measures provide a reliable and flexible foundation for solving decision-making problems involving vague, inconsistent, and periodic data, and offer notable advantages over existing fuzzy and neutrosophic models in scenarios where amplitude and phase information must be jointly considered. This work contributes to the growing body of research on complex neutrosophic structures and lays the groundwork for further development of aggregation operators, similarity measures, and application-driven extensions of Q-CFNS in fields such as pattern recognition, medical diagnosis, and multi-criteria decision making.

References

1. L.A. Zadeh, "*Fuzzy sets as a basis for a theory of possibility*", Fuzzy Sets and Systems, 2, 153-165, (1979).
2. Atanassov, K, "Intuitionistic Fuzzy Sets", Fuzzy Sets and Systems:20, 87-96, 1986.
3. F.Smarandache, A geometric interpretation of the neutrosophic set –"*A generalization of the intuitionistic fuzzy set*", Granur computing (GrC), IEEE International Conference,2011,pp.602-606, 2011.
4. Y. B. Jun. Q-Fuzzy subalgebras of BCK/BCI algebras, Scientiae Mathematicae Japonicae Online, vol. 4, pp. 197-302, 2001.
5. E.H Roh, K. H. Kim, J. G. Lee, Jetsonistic Q-fuzzy subalgebras of BCK/BCI-algebras, International Mathematical Forum, vol. 1, pp. 1167-1174, 2006
6. O.Kazanci, S. Yamuk, S. Yilmaz, On intuitionistic Q-fuzzy R-subgroups of near rings, International Mathematical Forum, vol. 2, pp. 2899-2910, 2007.
7. R. Muthuraj, P. M. Sithar Selvam, M. S. Muthuraman, Anti Q fuzzy group and its lower level subgroups. International Journal of Computer Applications, vol. 3, pp. 16-20, 2010
8. V. Lakshmana Gomathi Nayagam, R. Muthuraj, K. H. Manikandan, Intuitionistic Q-fuzzy HX group, International Journal of Mathematical Archive, vol. 2, pp. 1133-1139, 2011.
9. P M. Sithar Selvam, T. Priya, T. Ramachandran, K. T. Nagalakshmi, Anti Q-fuzzy R- closed KU-ideals in KU-algebras and its lower level cuts, International Journal of Furry Mathematical Archive, vol. 4, pp 61-71, 2014
10. K. Tanamoon, S. Sripaeng, A. Iampan, Q-Fuzzy sets in UP algebras, Songklanakarin Journal of Science and Technology, vol. 40, pp. 9-29, 2018

11. M. Abu Qumar, N. Hassan, An approach toward a Q-neutrosophic soft set and its application in decision making. Symmetry, vol. 11. pp. 139, 2019
12. S. Thiruveni, A. Solairaju, Neutrosophic Q-fuzzy subgroups, International Journal of Mathematics and its Applications, vol. 6, pp. 859-866, 2018
13. F. Adam, N. Hassan, Q-fuzzy soft set. Applied Mathematical Sciences, vol. 8, pp. 8689-8695, 2014.
14. S. Broumi. Q-Intuitionistic fuzzy soft sets, Journal of New Theory, vol. 5, pp. 80-91, 2015
15. T. Mahmood, Q. Khan, M. A. Khan, Q-Single valued neutrosophic soft sets, Journal of New Theory, vol. 13. pp. 10-25, 2016.
16. Al-Quran, Ashraf & Ahmad, Abd & Al-Sharqi, Faisal & Lutfi, Abdalwali. Q-Complex Neutrosophic Set, International Journal of Neutrosophic Science (IJNS) Vol. 20, No. 02, pp. 08-19, 2023.
17. Ramot, D., Milo,R., Friedman, M., & Kandel,A. "Complex Fuzzy Sets", IEEE Transactions and Fuzzy System, 10, 171-186.
18. Ali, Mumtaz, and Florentin Smarandache, "Complex neutrosophic set", Neural computing and applications, vol. 28, no.7, pp.1817-1834, 2017.
19. Alkouri, Abdulazeez (Moh'D. Jumah) S., and Abdul Razak Salleh, "Complex Intuitionistic fuzzy sets", AIP conference proceedings, American Institute of Physics, vol 1482, no.1, pp 464-470, 2012.
20. Antony, C.S.C., Jansi,R."Fermatean Neutrosophic Sets", International Journal of Advanced Research in Computer and Communication Engineering, 10(6), 24-27, 2021.
21. Broumi Said; Swaminathan Mohanaselvi; Tomasz Witzak; Mohamed Talea; Assia Bakali and Florentin Smarandache. "Complex Fermatean Neutrosophic Graph and Application to Decision Making": Decision Making: Applications in Management and Engineering, 2023.
22. Vijaya, V., Broumi, S., & Manikandan, H. "Complex Fermatean Neutrosophic Sets and their Applications in Decision Making". *Neutrosophic Sets & Systems*, *81,2025*.
23. Beaula Thangaraj, and V. Vijaya. "Critical Path in a project network using a new representation for Trapezoidal Fuzzy Numbers." *International Journal of Mathematics Research* 4.5 : 549-557,2012.
24. Beaula Thangaraj, and V. Vijaya. "A new method to find critical path from multiple paths in project networks." *Int J Fuzzy Math Arch* 9.2: 235-243, 2015.
25. Beaula, T., and V. Vijaya. "A Study on Exponential Fuzzy Numbers Using α- Cuts." *International Journal of Applied* 3.2: 1-13, 2013.
26. Vijaya, V., and D. Rajalaxmi. "Decision making in fuzzy environment using Pythagorean fuzzy numbers." *Mathematical Statistician and Engineering Applications* 71.4 : 846-854,2022.
27. Vijaya, V., D. Rajalaxmi, and H. Manikandan. "Finding critical path in a fuzzy project network using neutrosophic fuzzy number." *Advances and Applications in Mathematical Sciences* 21.10 :5743-5753,2022.
28. V, Vijaya. "Cubic Bipolar Complex Neutrosophic Sets in Medical Diagnosis: A New Paradigm". International Journal of Applied Mathematics. 38. 999-1015. 10.12732/ijam.v38i4s.284, 2025.
29. Vijaya V, Rajalakshmi G & Rajalaxmi D, "A Novel Framework of Q-Complex Fermatean Neutrosophic Sets with Algebraic and Operational Properties", Indian Journal of Natural Sciences, Volume XVII (17) (96) (2026) ISSUE 96115319-115328, 2026.