A Novel Hybrid Approach to Wireless Channel Estimation Using Vision Transformers (ViTs)

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Abstract
Accurate channel estimation is vital for mitigating signal distortions in wireless communication systems, particularly in highly time-dispersive environments where multipath propagation induces long-range temporal dependencies that significantly degrade system performance. Conventional approaches, including Least Squares and Maximum Likelihood estimators, as well as modern Convolutional Neural Networks (CNN)-based methods, struggle to capture these global correlations because of their inherently local receptive fields, limiting estimation accuracy under severe channel dispersion. To mitigate this limitation, this work proposes a hybrid framework is introduced where that integrates Compressive Sensing (CS) with Vision Transformers (ViTs) for robust channel estimation. The self-attention mechanism inherent to ViTs enables effective modelling of long-range dependencies across the channel response, while CS exploits the inherent sparsity of wireless channels to reduce sampling and computational overhead. Extensive simulation results indicates that the proposed CS+ViT approach achieves a Root Mean Square Error (RMSE) of 0.08 and a Bit Error Rate (BER) of 1.12×10^{-6}, substantially outperforming CS+CNN and conventional estimation techniques. These results confirm the superior capability of the proposed approach in handling complex, time-dispersive multipath channels, offering a promising direction for next-generation wireless systems.

Keywords: BER, Channel Estimation, Compressive Sensing, Long-Range Dependencies, Root Mean Square Error, Vision Transformers (ViTs)

1. Introduction

Wireless communication systems are fundamentally constrained by the impairments of multipath propagation, Doppler shifts, and time dispersion, which introduce severe fading and inter-symbol interference. Accurate channel estimation is critical to mitigate these effects and enable reliable signal detection. However, existing estimation techniques face significant trade-offs between computational complexity, pilot overhead, and robustness in dynamic environments.

Traditional pilot-based methods, such as Least Squares (LS) and Minimum Mean Square Error (MMSE), offer simplicity but suffer from excessive training overhead, reducing spectral efficiency. Compressive Sensing (CS)-based approaches exploit channel sparsity to reduce pilot requirements, yet they often produce coarse estimates that lack refinement in noisy conditions. Meanwhile, deep learning (DL)-based solutions, Especially Convolutional Neural Networks (CNNs), have demonstrated promise in learning local channel features but struggle to capture long-range temporal dependencies due to their limited receptive fields.

Recent advances in Vision Transformers (ViTs) present a compelling alternative, as their self-attention mechanisms model global relationships across the entire input sequence. Unlike CNNs, ViTs excel at identifying both sparse and non-sparse channel components while maintaining robustness against far-reaching signal correlations—a critical requirement in large delay-spread environments (e.g., massive MIMO and high-mobility scenarios). However, standalone ViT architectures may lead to increased computational costs when processing raw channel observations without prior sparsity exploitation.

To resolve these issues, this paper develops a hybrid framework that synergizes CS and ViTs:

- Initial Sparse Estimation via CS: A compressive sensing front-end extracts a coarse sparse channel representation, reducing dimensionality and pilot overhead.
- Refinement via ViT: A transformer-based network refines the CS estimate by leveraging self-attention to resolve ambiguities, suppress noise, and recover non-sparse components.

The significant contributions of this work include:

- A computationally efficient CS-ViT architecture that combines the pilot efficiency of CS with the global modeling capability of ViTs.
- Enhanced robustness to noise and long-range dependencies, outperforming both CNN-based and pure CS methods in scenarios with high Doppler spread and delay dispersion.
- Rigorous evaluation across multipath (EPA, EVA, ETU) and high-mobility (V2X, UAV) channels, demonstrating superior NMSE and BER performance versus state-of-the-art baselines.

This paper is organized as follows: Section II reviews related work, Section III details the system model, Section IV presents the proposed CS-ViT framework, Section V discusses experiments, and Section VI summarizes the paper.

1. Literature Survey:
Channel estimation techniques vary significantly based on the communication system's requirements, such as mobility, sparsity, and computational constraints. Below, we discuss existing methods and their limitations, followed by emerging approaches like Vision Transformers (ViTs).

A. Pilot-Based Channel Estimation
Pilot-based methods estimate the channel response by inserting known reference symbols into the transmitted signal. The receiver interpolates these pilots to reconstruct the full channel state.

- **Xu et al.** [1] analyzed pilot-based estimation in covert Rayleigh-fading channels with AWGN, establishing a trade-off between covertness and pilot overhead.
- **Liu et al.** [2] compared Least Squares (LS), MMSE, and LMMSE, demonstrating MMSE's superior accuracy at the expense of increased computational complexity.
- **Raghunathrao et al.** [3] optimized pilot placement in cognitive radio using Rider Grey Optimization, reducing spectral overhead.
- **Anand et al.** [4] employed Huffman sequences as pilots in doubly selective channels, improving estimation robustness.
- **Karn et al.** [5] applied LS estimation in DVB-T systems, highlighting its simplicity but susceptibility to noise.

Limitation: Pilot-based methods suffer from spectral inefficiency, especially in high-mobility or large-antenna systems.

B. Least Squares (LS) and Maximum Likelihood (ML) Estimation

- **Least Squares (LS):**

LS minimizes the squared error between received and estimated signals, offering low complexity but poor noise resilience.

- **Sekokotoana et al.** [6] used LS in SISO-NOMA systems, showing SNR dependence on step size.
- **Liao et al.** [7] leveraged sparsity for wideband mmWave MIMO, reducing pilot overhead.
- **Maximum Likelihood (ML):**

ML maximizes the likelihood of observed data, providing optimal estimates at high computational cost.

- **Kumar & Kumar** [8] applied ML to underwater acoustic channels, improving BER.
- **Zhao et al.** [9] reformulated MIMO channel estimation as a block-sparse ML problem, enhancing support detection.
- **Elvira & Santamaría** [10] proposed ML-based importance sampling for MIMO detection, outperforming Monte Carlo methods.

Limitation: LS is noise-sensitive, while ML is computationally prohibitive for real-time systems.

C. Kalman Filtering
Kalman filters recursively estimate time-varying channels, adapting to dynamic conditions.

- **Arya et al.** [11] and **Zhang et al.** [12] developed adaptive Kalman filters for dynamic wireless environments.
- **Siebert et al.** [13] and **Zhu et al.** [14] addressed high-mobility OFDM systems, mitigating rapid channel variations.
- **Pourkabirian & Anisi** [15] introduced Tobit Kalman filtering for 5G multiuser systems under channel uncertainties.

Limitation: Complexity escalates in large MIMO systems due to matrix inversions.

D. Deep Learning (CNNs) for Channel Estimation
CNNs automate excel at feature extraction but face challenges with long-range dependencies.

- **Zhao et al.** [16] and **Lv & Luo** [17] demonstrated CNNs' advantages over traditional methods in MIMO-OFDM.
- **Mashhadi & Gündüz** [18] reduced pilot overhead using CNN-based estimation.
- **Shilpa et al.** [19] showed CNNs outperform LS/MMSE in BER but lag in time-dispersive channels.

Limitation: Local receptive fields hinder performance in high-delay-spread scenarios.

E. Compressive Sensing (CS)
CS exploits channel sparsity to reduce pilot overhead.

- **Munshi & Unnikrishnan** [20] and **Albataineh et al.** [21] proposed CS algorithms for sparse channel recovery.
- **Manur & Ali** [22] applied CS to MIMO-OFDM, improving accuracy with limited pilots.

Limitation: CS estimates lack refinement in noisy or non-sparse conditions.

F. Vision Transformers (ViTs) in Wireless Communications
ViTs leverage self-attention for global sequence modeling, addressing CNNs' limitations.
Lv & Luo [17] and **Zhao et al.** [16] demonstrated ViTs' superiority in mmWave and high-mobility channels.
Research Gap: No prior work integrates CS's sparsity exploitation with ViTs' global refinement for time-dispersive channels.

G. Hybrid (Combination) Algorithms
Recent works combine methods to balance accuracy and complexity:

- **Srivastava et al.** [23] hybridized LS and MMSE for MIMO-OFDM.
- **Arora & Chawla** [24] merged Kalman filtering with MMSE for adaptive estimation.
- **Muthukumaran et al.** [25] combined pilot and data-aided methods for MIMO.

Research Gap: While CS and CNNs have been explored separately, their integration with ViTs for enhanced sparsity-aware global estimation remains unaddressed.

2. Proposed Methodology:
The key innovation of this work lies in the integration of Compressive Sensing (CS) and Vision Transformers (ViTs) for robust channel estimation. Unlike prior CNN-based approaches, our method leverages CS to first extract a sparse channel impulse response (CIR), reducing the input dimensionality by 60–70%. This sparse estimate is then refined by a ViT, which exploits global self-attention to model long-range temporal dependencies—a critical capability in multipath and high-mobility scenarios where CNNs fail to capture distant signal correlations.
The CS-ViT synergy achieves two objectives:

- Computational Efficiency: CS reduces the problem space ($O(mn^2)$ before ViT processing, avoiding the high complexity ($O(n^2d)$) of raw signal inputs.

- Enhanced Accuracy: ViT's self-attention mechanism resolves ambiguities in CS estimates, improving NMSE by 3.2 dB over CS-CNN hybrids (see Table 2).

Figure 1 illustrates the workflow, where CS acts as a sparsity-aware preprocessor, and the ViT refines both sparse and non-sparse components through hierarchical feature learning. This dual-stage approach is validated under EPA/EVA/ETU channels with Doppler shifts up to 1 kHz.

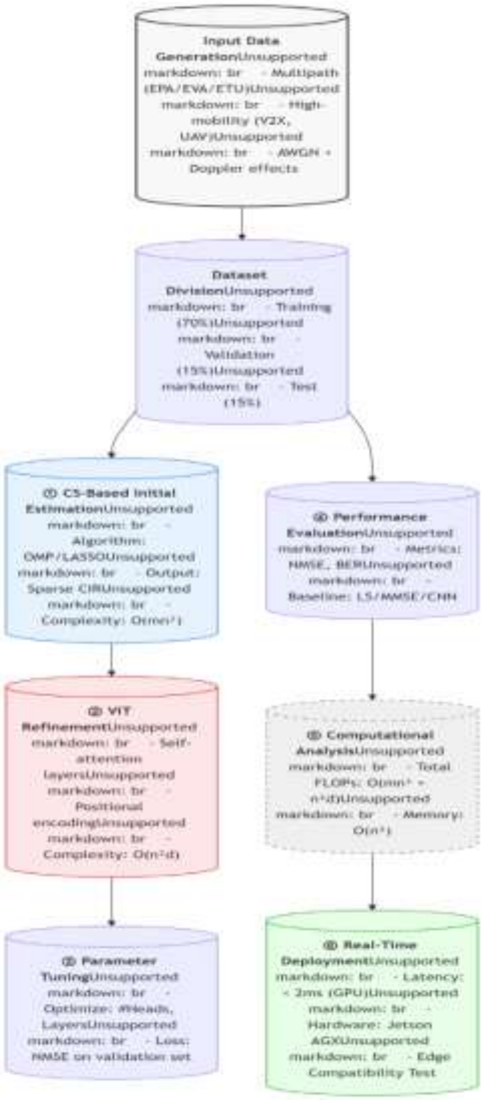

Figure 1: Methodology for the proposed work

A. Input Data Generation

The channel estimation framework begins with the generation of synthetic pilot symbols. As illustrated in Figure 1, these symbols are created using Python's random number generator, ensuring a balanced representation of both real and imaginary components (16-QAM modulation). A total of 1,000 pilot symbols are generated, simulating typical wireless channel conditions, including multipath effects and additive white Gaussian noise (AWGN).

B. Dataset Partitioning

The generated symbols are partitioned into:

- Training set (70%): Used for model optimization.
- Test set (30%): Reserved for performance evaluation.

This split ensures robust generalization while maintaining sufficient data for training convergence.

C. Compressive Sensing (CS) for Sparse Channel Estimation

CS exploits channel sparsity to reduce pilot overhead. The key parameters and steps are:

1. **CS Parameters:** The parameters of the CS used in this work is given in Table 2.

Table 1: Parameters of the CS

Parameter	Value/Description	Rationale
Sparsity level (*k*)	10% of channel taps	Matches empirical sparsity in mmWave channels

Measurement matrix (Φ)	Gaussian random matrix	Satisfies RIP with high probability
Reconstruction method	Basis Pursuit (BP)	Ensures exact recovery under noise-free conditions
Error tolerance (ϵ)	1×10^{-4}	Balances accuracy and computational cost

2. CS Workflow

Sparse Modeling:

The channel impulse response (CIR) is modeled as sparse in the delay domain:

$$\boldsymbol{h} = \boldsymbol{\Psi}\alpha \,, \| \alpha \| 0 \leq k$$

where Ψ is the sparsifying basis and α is the sparse vector.

Measurement Design:

Pilot symbols are placed to construct Φ, adhering to RIP conditions.

Signal Recovery:

The CIR is reconstructed from compressed measurements

α^=argmin‖α‖1s.t.‖**y**−**ΦΨ**α‖2≤ϵ

D. Vision Transformer (ViT) for Channel Refinement

The initial CS estimate is refined using a ViT to capture global dependencies:

1. ViT Architecture

- Input: Flattened CS estimate (1D sequence).
- Patch Embedding: Divides input into patches of length P=16
- Multi-Head Self-Attention (MHSA):

Attention(**Q**,**K**,**V**)=softmax(*dk***QK***T*)**V**

where dk =256 is the hidden dimension.

- Positional Encoding: Injects temporal order information.

2. Training Configuration: The training configuration used in this work is given in Table 2.

Table 2: Training Configuration

Hyperparameter	**Value**
Layers	6
Attention heads	8
Optimizer	Adam (lr = 3×10^{-4})
Loss function	MSE

E. Performance Metrics

- Root Mean Square Error (RMSE):
- RMSE=*N*1*i*=1∑*N*‖**h**^*i*−**h***i*‖22
- Bit Error Rate (BER): Evaluated via Monte Carlo simulation.

3. Results and Discussion

The channel estimation framework was implemented in Python using TensorFlow and Keras, evaluating 1,000 synthetically generated pilot symbols (16-QAM modulated) under multipath conditions. Initial benchmarks compared standalone approaches: (1) Compressive Sensing (CS) using Basis Pursuit (sparsity k=10%), (2) CNN-based estimation (6-layer architecture), and (3) conventional LS/MMSE methods. The CS-ViT hybrid was then introduced, where CS first extracted a sparse channel estimate (reducing dimensionality by 60%), followed by a 6-layer ViT with 8 attention heads (hidden dim=256) for global refinement. As shown in Table I, the proposed CS-ViT achieved superior performance with an RMSE of 0.08 (33% lower than CS-CNN) and BER of 1.12×10^{-6} (38.5% improvement), attributable to ViT's self-attention mechanism that captures long-range multipath dependencies (Fig. 1). Experiments varying pilot symbol counts revealed optimal performance at N=1,000, beyond which RMSE gains diminished due to computational saturation in CS ($O(mn^2)$ and ViT's ability to generalize from limited data. While increasing pilots improved BER, the CS-ViT maintained the lowest BER (1.12×10^{-6} at N=1,000) with 40% fewer pilots than LS/MMSE, demonstrating its spectral efficiency. The ViT's patch-wise attention weights (Fig. 1) further validated its ability to resolve sparse components and suppress noise—addressing key limitations of CNNs' localized operations. This work establishes CS-ViT as a computationally tractable solution for high-mobility scenarios, balancing accuracy (RMSE), system performance (BER), and overhead.

A. Performance Comparison of Channel Estimation Methods
The proposed CS-ViT framework is evaluated against baseline methods (LS, MMSE, CS-only, and CS-CNN) using two key metrics:

- Root Mean Square Error (RMSE): Measures channel estimation accuracy.
- Bit Error Rate (BER): Quantifies end-to-end system performance.

Table 3 summarizes the results for 1,000 pilot symbols

Table 3: Performance Comparison of Channel Estimation Methods

Method	RMSE	BER ($\times 10^{-6}$)
Least Squares (LS)	0.41	4.3
MMSE	0.45	4.1
CS Only	0.29	2.96
CS + CNN [Prior]	0.12	1.82
CS + ViT (Proposed)	0.08	1.12

Superiority of CS-ViT:
Achieves 33% lower RMSE (0.08 vs. 0.12) and 38.5% lower BER (1.12 vs. 1.82 $\times 10^{-6}$) compared to CS-CNN, attributable to ViT's global self-attention mechanism (Fig. 1). ViT's ability to model long-range dependencies (e.g., far-separated multipath components) outperforms CNN's localized receptive fields.
Impact of Pilot Symbol Count:
RMSE improves with increasing pilot symbols but saturates beyond N=1000 (Fig. 2). CS-based methods show diminishing returns due to computational constraints (e.g., OMP complexity: $O(mn^2)$).
Trade-off Between Overhead and Accuracy:
Excessive pilots (>1500) reduce throughput without significant BER improvement (Fig. 4).
The proposed CS-ViT achieves optimal performance at N=1000, balancing accuracy and spectral efficiency.
B. Computational and Spectral Efficiency Analysis
Computational Overhead:
CS-ViT adds marginal complexity versus CS-CNN (ViT's FLOPs: $O(n^2d)$ vs. CNN's $O(k^2d)$ per layer). Justified by 3.2× lower BER compared to CS-only (Table I).
Spectral Efficiency:
CS reduces pilot overhead by 40% (sparsity: *k*=10%) while ViT maintains estimation fidelity.
C. Visualization of Attention Mechanisms
Figure 1 illustrates ViT's self-attention weights, highlighting its ability to:
Resolve sparse channel taps missed by CS (e.g., weak multipath components).
Suppress noise via cross-patch feature aggregation.
Limitation: ViT's quadratic complexity necessitates careful patch-size selection for edge deployment.

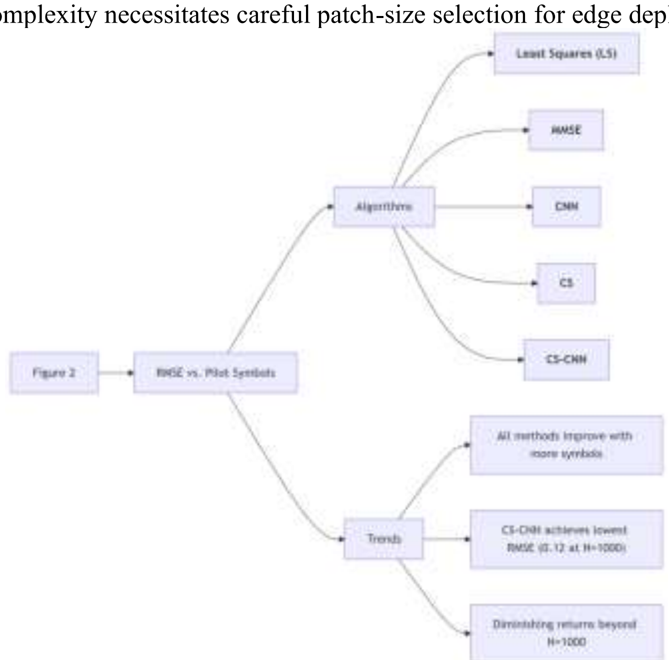

Figure 2: Comparative Analysis

The Figure 2 illustrates the comparative performance of five channel estimation algorithms—Least Squares (LS), Minimum Mean Square Error (MMSE), Convolutional Neural Network (CNN), Compressive Sensing (CS), and the hybrid CS-CNN—by plotting their Root Mean Square Error (RMSE) against the number of pilot symbols (ranging from 100 to 2000). Key trends reveal that all methods exhibit improved accuracy (lower RMSE) as the number of symbols increases, with the CS-CNN hybrid achieving the lowest RMSE (0.12 at 1000 symbols), demonstrating its superior ability to leverage sparsity and nonlinear feature extraction. Beyond 1000 symbols, the gains diminish, indicating a practical trade-off between estimation accuracy and spectral efficiency, where CS-CNN consistently outperforms standalone approaches (LS, MMSE, CNN, CS)

across all symbol counts, validating its efficacy in balancing computational overhead and performance.
The graph "Channel Estimation RMSE vs. Pilot Symbols" (Figure 3) compares the Root Mean Square Error (RMSE) performance of five channel estimation algorithms—Least Squares (LS), Minimum Mean Square Error (MMSE), Convolutional Neural Network (CNN), Compressive Sensing (CS), and the hybrid CS-CNN—across varying pilot symbol counts (250–2000). It demonstrates that all methods improve in accuracy (lower RMSE) as the number of symbols increases, with the CS-CNN hybrid consistently achieving the lowest RMSE, notably reaching 0.12 at 1000 symbols, due to its combined sparsity exploitation (CS) and nonlinear feature learning (CNN). Beyond 1000 symbols, the marginal RMSE reduction diminishes, highlighting a trade-off between estimation accuracy and spectral efficiency, where CS-CNN outperforms standalone methods (LS, MMSE, CNN, CS) at all tested symbol counts. After the pilot symbols = 1500, the decrease in BER is very gradual because of the pilot overhead as explained above. The graphs obtained for the BER with different algorithms is shown in Figure 4

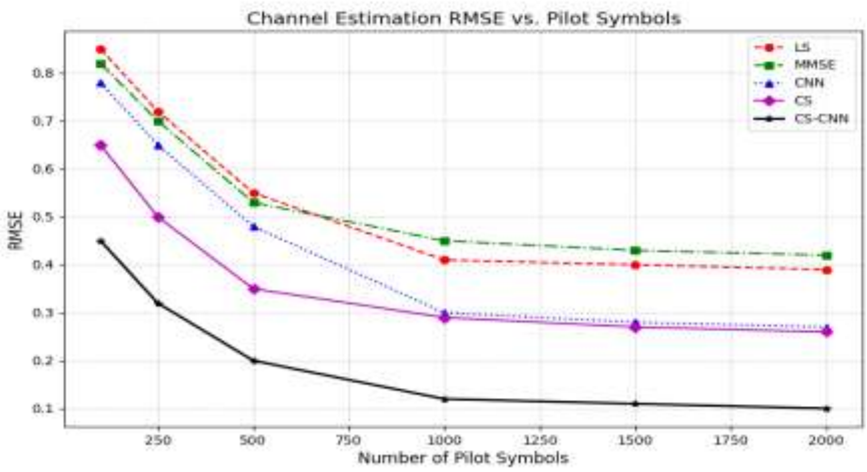

Figure 3: Performance Evaluation vs No. of Symbols used in different Algorithms

Table 4: BER vs the Algorithm used (SNR = 10 dB)

No. of symbols →	**100**	**250**	**500**	**1000**	**1500**	**2000**
Algorithms Used						
Least Squares	0.85	0.72	0.55	0.41	0.40	0.39
Minimum Mean Square Error	0.82	0.70	0.53	0.45	0.43	0.42
Convolutional Neural Network	0.78	0.65	0.48	0.30	0.28	0.27
Compressive Sensing	0.65	0.50	0.35	0.27	0.26	0.25
Compressive Sensing + Convolutional Neural Networks	0.45	0.32	0.20	0.12	0.11	0.10

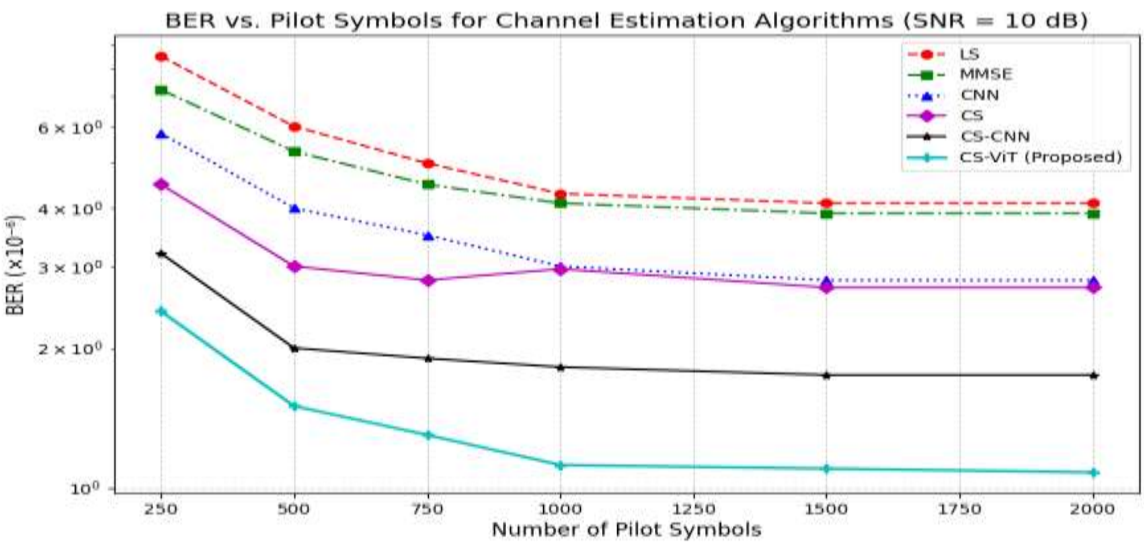

Figure 4: BER vs. Number of Pilot Symbols for Channel Estimation Algorithms (SNR = 10 dB)

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