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Mathematical Modeling for Crop Allocation in Agricultural Fields

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Abstract

This study presents a Picture Fuzzy Assignment Problem (PFAP) for optimally assigning four distinct crops to four equal-sized paddocks with varying soil fertility. Since fertilizer requirements are uncertain due to changing agricultural conditions and expert judgment, picture fuzzy numbers are used to represent the associated costs. The objective is to determine the one-to-one crop–paddock assignment that minimizes the total fertilizer cost. The proposed picture fuzzy approach effectively handles uncertainty and provides a reliable decision-making framework for achieving cost-efficient and sustainable crop allocation.

Introduction

Within mathematics, operations research is a significant field. Numerous real-world applications exist for it, such as in decision-making, transportation issues, linear programming issues, and more. Economics as a framework for techniques and concepts is used to analyze problems in various domain. Innovative production techniques are required to meet the expanding global population's growing demand. Farmers deal with issues related to the best number of plants to plant on their farm, the distribution of labour among fields, and the application of fertilizer. Allocation issues also arise in other academic domains, such as salesperson to area, operator to machines, and product to product mix. Allocation issues also arise in other fields of study, such as salesperson to area, operator to machines, and product to product mix. Indeed, all academic disciplines are conducive to allocation problems, wherein the distribution of resources necessitates their optimal utilization to accomplish the objective(s) of the organization. Experiments conducted in the field are a common method of research in the agriculture field to understand the many factors that affect produce quality and productivity. The most important component of field-driven agricultural experiments that can be studied with the use of mathematical programming is planning. An extremely helpful subset of mathematical programming called assignment problems is used to analyze issues in agriculture. Assignment problems present a special configuration in which issues related to resource allocation are interpreted as a matrix of jobs to be assigned to the numbers of workers. It is ensured that the objective function is minimized (cost) or maximized (profit or sales) while doing this. The framework for assignment problems includes several popular techniques that provide solutions, such as the Hungarian method, neural networks, genetic algorithms, etc. The ultimate goal of the
assignment problem is to ascertain the best possible distribution of jobs (origins) among the same number of people (destinations), with the goal of minimizing or maximizing the allocation problem's overall cost or profit. Many algorithms and methods, such as the Hungarian method, neural networks, genetic algorithms, etc., have been developed in order to obtain the optimal allocation of assignment problems.

Nowadays fuzzy sets plays a vital role in all the fields like topology, graph theory etc. The combination of operations research and fuzzy sets solved many practical problems. Fuzzy set is a developed field. Many of the extensions still going in the fuzzy set. It is extended in an Intuitionistic fuzzy set, Pythagorean fuzzy set, Neturosophic fuzzy set, Picture fuzzy set etc. Comparing with one another the covering area of the solution is higher. That is the main advantage in the fuzzy set.

Picture fuzzy set (PFS) [6] is an extension of fuzzy set (FS) [19] and intuitionistic fuzzy set (IFS) [2, 7, 8, 12,13] and by providing the neutral and refusal membership degrees, it can readily handle the ambiguity inherent in human mental processes. In order to address uncertainties greater than intuitionistic fuzzy uncertainties, picture fuzzy sets have been proposed as a generalization of intuitionistic fuzzy sets. Memis. S proposed to maintain continuity, this idea and a few of its procedures have been changed [14]. The neutral membership degree and the refusal membership degree are the two components of the hesitation margin of IFS that the authors separated in PFS. The PFS reverts to IFS when the neutral and refusal membership degrees are both zero, or when the hesitation margin hits zero. It can be challenging for FS and IFS to articulate instances in which human thought involves more than one option, such as "yes," "abstain," "no," and "refusal." When handling cases of this nature, PFS is the better option because it offers positive, neutral, negative, and refusal membership degrees. One instance of a situation where voters can express their ideas is during a national general election, where they can "vote for," "abstain," "vote against," and "refuse" to participate in the election [4, 5].

There is one candidate running for office and one thousand voters. Out of them, 400 voters support the candidate, 100 choose not to vote, meaning they will continue to abstain from the process, 300 vote against the candidate, and 200 voters refuse to support the candidate, meaning they would vote for NOTA. Since FS and IFS do not support neutrality, situations like this one are possible in real life and fall outside of their purview. As an additional illustration, let's say a specialist solicits a person's opinion on a particular item. The individual can now state that there are four possible outcomes: 0.4 is the object is good, 0.3 is the object is not good, 0.2 is the object is both good and bad, and 0.1 is the probability that (s) he does not know about the object exists. Neither the FSs nor the IFSs are in charge of this problem. Because PFS can accommodate additional viewpoints, it has grown to be a valuable tool for handling vague and imprecise information. Some researchers have used PFS to solve numerous real-world issues [20, 4, 5]. Hungarian method was proposed by Kuhn [11], it is utilized primarily for resolving assignment-related issues. Fatma Kutlu Gundogdu [9] expanding upon the traditional linear assignment approach, a new method known as the picture fuzzy linear assignment method (PF-LAM) has been developed to address multiple criteria group decision-making problems using picture fuzzy sets. On solving transportation problems that involve fully picture fuzzy (FPFTPs) data, these problems are formulated using picture fuzzy linear programming which was proposed by Muhammad Athar Mehmood [16]. Amalendu si etal [1] was proposed a novel method to compare the PFNs even when the score and accuracy values are equal. A multi-attribute decision-making problem was introduced [3] and solved by using harmonic mean aggregation operators with trapezoidal fuzzy number in picture fuzzy environment. Dijkstra algorithm was proposed [10] in picture fuzzy shortest path problem, in which trapezoidal picture fuzzy numbers represent the costs incurred by the arcs. Mushtaq. A.Lone [15] was proposed Branch – Bound method to found optimal solution in intuitionistic fuzzy assignment problem and defuzzification done by using accuracy function. Saifullah Khan etal introduced a logarithmic decision-making approach to deal with uncertainty in the form of a picture fuzzy set [17]. Sriyani violin etal solved job assignment problem by using Branch – Bound method [18].

Contributions of the Paper

- A Branch-and-Bound framework is proposed for solving Picture Fuzzy Assignment Problems.
- Trapezoidal picture fuzzy numbers are transformed into crisp values using the accuracy function.
- The proposed method solves minimization, maximization, and unbalanced assignment problems.
- The computational complexity of the proposed method is analyzed.
- The proposed approach reduces computational effort through pruning strategies while preserving optimality.

Research Gap

Existing studies mainly focus on Hungarian methods and ranking approaches for solving fuzzy assignment problems. However, limited research has been carried out on solving Picture Fuzzy Assignment Problems using exact optimization techniques such as the Branch-and-Bound method. Therefore, this paper proposes a Branch-and-Bound framework to obtain optimal solutions for trapezoidal picture fuzzy assignment problems under uncertain environments.

2. Preliminaries

2.1 Picture Fuzzy Set

A Picture Fuzzy Set (PFS) B on the universe V is an object in the form of

B = $\{(\overset{*}{x},\overset{*}{\alpha}(x),\overset{*}{\beta}(x),\overset{*}{\delta}(x))/x\in V\}$ Where $\overset{*}{\alpha}(x)\in[0,1]$ be the degree of positive membership of $\overset{*}{x}$ in B, similarly $\overset{*}{\beta}(x)\in[0,1]$ and $\overset{*}{\delta}(x)\in[0,1]$ are respectively called the degree of neutral and negative membership of $\overset{*}{x}$ in B. The parameters $\{\overset{*}{\alpha}(x),\overset{*}{\beta}(x),\overset{*}{\delta}(x)\}$ satisfy the following condition.

$$0\le\overset{*}{\alpha}(x)+\overset{*}{\beta}(x)+\overset{*}{\delta}(x)\le 1,\forall x\in V$$

The refusal membership degree $\overset{*}{\varphi}(x)$ of $\overset{*}{x}$ in B can be calculated appropriately,

$$\overset{*}{\varphi}(x)=1-\{\overset{*}{\alpha}(x)+\overset{*}{\beta}(x)+\overset{*}{\delta}(x)\},\forall \overset{*}{x}\in V$$

For a fixed $\overset{*}{x}\in V$,, $\{\overset{*}{\alpha}(x),\overset{*}{\beta}(x),\overset{*}{\delta}(x)\}$ is called picture fuzzy number, Where $\overset{*}{\alpha}(x)\in[0,1]$ $\overset{*}{\beta}(x)\in[0,1]$, $\overset{*}{\delta}(x)\in[0,1]$, $\overset{*}{\varphi}(x)\in[0,1]$ and $\overset{*}{\alpha}(x)+\overset{*}{\beta}(x)+\overset{*}{\delta}(x)+\overset{*}{\varphi}(x)=1$.

2.2. Picture Fuzzy Number

A Picture fuzzy number N in the set of real numbers V can be defined as a picture fuzzy set N ={ $(\overset{*}{x},\overset{*}{\alpha}(x),\overset{*}{\beta}(x),\overset{*}{\delta}(x))/x\in V$ },where

$$\hat{\alpha}(x)=\begin{cases} I^{l}(\overset{*}{x}), m_1\le x<n \\ \lambda, n\le x<o \\ I^{r}(\overset{*}{x}), o<x\le p_1 \\ 0, otherwise \end{cases}$$

$$\hat{\beta}(x) = \begin{cases} J^l(\overset{*}{x}), m_2 \le x < n \\ \gamma, n \le x < o \\ J^r(\overset{*}{x}), o < x \le p_2 \\ 0, otherwise \end{cases}$$

$$\hat{\delta}(x) = \begin{cases} K^l(\overset{*}{x}), m_3 \le x < n \\ \nu, n \le x < o \\ K^r(\overset{*}{x}), o < x \le p_3 \\ 0, otherwise \end{cases}$$

2.3. Trapezoidal picture fuzzy numbers

A trapezoidal picture fuzzy number for every element $\overset{*}{x}$, associated with three membership functions . In general, when

$$I^l(\overset{*}{x}) = \frac{x - m_1}{n - m_1}\lambda^n, \qquad I^r(\overset{*}{x}) = \frac{p_1 - x}{p_1 - o}\lambda$$

$$J^l(\overset{*}{x}) = \frac{n - x + \gamma(x - m_2)}{n - m_2}, \qquad J^r(\overset{*}{x}) = \frac{x - o + \gamma(p_2 - x)}{p_2 - o}$$

$$K^l(\overset{*}{x}) = \frac{n - x + \nu(x - m_3)}{n - m_3}, \qquad K^r(\overset{*}{x}) = \frac{x - o + \nu(p_3 - x)}{p_3 - o}$$

For $0 \le \lambda, \gamma, \nu \le 1$ satisfying $\lambda + \gamma + \nu \le 1$, the picture fuzzy number is called trapezoidal picture fuzzy number.

2.4. Expected Value

The expected value of Trapezoidal picture fuzzy number N $=\{(m,n,o,p)\ \lambda, \gamma, \nu\}$ is defined as

$$I\,\mathrm{N} = \frac{1}{8}[(m+n+o+p) \times (1 + \lambda - \gamma - \nu)]$$

2.5. Score Function

Let N $=\{(m,n,o,p)\ \lambda, \gamma, \nu\}$ be a Trapezoidal picture fuzzy number then,

S (N) = I N $\times(\lambda - \gamma - \nu)$ is named as score function,

where I N is the expected value of N and S (N) $\in$ [-1,1].

2.6. Accuracy Function

Let N $=\{(m,n,o,p)\ \lambda, \gamma, \nu\}$ be a Trapezoidal picture fuzzy number then,

H (N) = I N $\times(\lambda + \gamma + \nu)$ is called accuracy function of N, where I N is the expected value of N and H(N) $\in$ [0,1].

2.7. Picture Fuzzy Assignment Problem

Assume that there are M tasks that need to be completed and M people are available to perform these tasks. Assume that everyone is capable of performing each task at hand, albeit in a different way.

Consider C $_{PQ}$ is a picture fuzzy cost of assigning P th person to the Q th task. Here X $_{PQ}$ denoting the decision variable of assigning P th person to the Q th job. Finding an assignment, or which job should be assigned to each individual on a one– to-one basis to minimize the overall cost of completing all tasks, is the challenge. These kinds of issues are referred to as assignment problems.

Mathematically as Picture Fuzzy Assignment Problem is given below

$$\text{Minimize / Maximize } Z = \sum_{\mathcal{P}=1}^{\mathcal{M}} \sum_{\mathcal{Q}=1}^{\mathcal{M}} \mathcal{C}_{\mathcal{P}\mathcal{Q}}\ \mathcal{X}_{\mathcal{P}\mathcal{Q}}$$

$$\text{Subject to } \sum_{\mathcal{Q}=1}^{\mathcal{M}} \mathcal{X}_{\mathcal{PQ}} = 1 \text{ for P =1,2,... } \mathcal{M}$$

$$\sum_{\mathcal{P}=1}^{\mathcal{M}} \mathcal{X}_{\mathcal{PQ}} = 1 \text{ for Q =1,2,... } \mathcal{M}$$

Where X_{PQ} = 0 or 1

Proposed Algorithm

* From the given data form an assignment cost table.
* Apply the accuracy function, to convert Trapezoidal picture fuzzy number into the Crisp value.
* Let 'M' ' be the level number in the branch tree, σ be the assignment in the current node of a branching tree. Assume that the root node is 0.
* $P_{\sigma}^{\mathcal{M}}$ be an assignment at level M' of the branching tree. $\mathcal{A}$ is the set of assigned cells up to the node $P_{\sigma}^{\mathcal{M}}$ from the root node (set i , j values with respect to the assigned cells up to the node $P_{\sigma}^{\mathcal{M}}$ from the root node). $\mathcal{V}_{\sigma}$ be the lower (upper) bound of the partial assignment up to $P_{\sigma}^{\mathcal{M}}$ such that,

$$\mathcal{V}_{\sigma} = \sum_{i,j\in\mathcal{A}} \mathcal{C}_{i,j} + \sum_{i\in\mathcal{X}} \Big(\sum_{j\in\mathcal{Y}} min\,\mathcal{C}_{i,j} \Big)$$

Where $\mathcal{C}_{i,j}$ is the cost value with respect to the i^{th} row and j^{th} column. $\mathcal{A}$ be the set of rows which are not deleted up to the node $P_{\sigma}^{\mathcal{M}}$ from the node in the branching node.
* At level M', the column marked as M' of the assignment problem, will be assigned with the best row of the assignment problem.
* The terminal node at the lower (higher) most level should be taken into consideration for additional branching if there are ties on the lower (upper) bound.
* If the minimum (maximum upper) lower bound happens to be at any one of the terminal nodes at the $(n-1)^{th}$ level, the optimality is reached. Subsequently, the tasks along the route from the root node to the specified node, in conjunction with the absent pair of row and column combinations, will constitute the optimal solution.

Optimality Theorem
Theorem
The proposed Branch-and-Bound algorithm always converges to the global optimal solution of the Picture Fuzzy Assignment Problem.

Proof
The proposed algorithm systematically explores feasible assignments while pruning only those branches whose lower bounds exceed the current best solution for minimization problems or whose upper bounds are smaller than the current best solution for maximization problems. Therefore, no feasible optimal branch is discarded during the search process. Hence, the obtained solution is globally optimal.

3. Numerical Example
3.1. Minimization Problem
Ponder about the Picture Fuzzy Assignment Problem, in which a farmer plans to sow four distinct crops in four paddocks of the same size. Four equal-sized paddocks, P1, P2, P3, and P4, are represented along columns, and four distinct crops, C1, C2, C3, and C4 are represented along rows. Varying crops have various nutrient requirements, and soil fertility varies throughout paddocks. Thus, the crop that is grown in which paddock will determine the overall cost of fertilizer that needs to be administered. Find the best strategy to distribute paddocks to crops such that the overall cost of fertilizer is as low as possible.

	P1,	P2,	P3	P4
C1	(2,5,7,9) (0.6,0.1,0.3)	(8,10,12,14) (0.5,0.25,0.1)	(4,6,8,10) (0.4,0.3,0)	(7,8,9,10) (0.55,0.35,0.15)
C2	(2,3,4,6) (0.6,0.1,0.2)	(10,11,14,15) (0.5,0.4,0)	(6,9,13,15) (0.6,0.0.4)	(1,3,5,7) (0.5,0.1,0.2)
C3	(12,13,14,15) (0.3,0.1,0.15)	(9,12,15,18) (0.45,0.15,0.1)	(1,2,3,4) (0.5,0.4,0)	(7,9,11,13) (0.55,0.4,0)
C4	(5,9,13,17) (0.6,0.1,0.2)	(8,11,14,17) (0.7,0.2,0.1)	(9,13,14,17) (0.6,0.2,0.1)	(14,15,16,17) (0.5,0.35,0)

Here Cost matrix (C_{PQ}) are considered as Trapezoidal Picture Fuzzy Number.

Solution
By using Accuracy function we get the following cost matrix table

$$\begin{bmatrix} 3.45 & 5.3762 & 2.695 & 4.6856 \\ 2.1937 & 6.1875 & 6.45 & 1.92 \\ 3.8981 & 5.67 & 1.2375 & 5.4625 \\ 6.435 & 8.75 & 7.7512 & 7.5756 \end{bmatrix}$$

The problem can be rewrite as follows

$$\begin{aligned} Min\ \ Z\ = \ & 3.45\mathcal{X}_{11} + 5.3762\mathcal{X}_{12} + 2.695\mathcal{X}_{13} + 4.6856\mathcal{X}_{14} + \\ & 2.1937\mathcal{X}_{21} + 6.1875\mathcal{X}_{22} + 6.45\mathcal{X}_{23} + 1.92\mathcal{X}_{24} + \\ & 3.8981\mathcal{X}_{31} + 5.67\mathcal{X}_{32} + 1.2375\mathcal{X}_{33} + 5.4625\mathcal{X}_{34} + \\ & 6.435\mathcal{X}_{41} + 8.75\mathcal{X}_{42} + 7.7512\mathcal{X}_{43} + 7.5756\mathcal{X}_{44} \end{aligned}$$

Subject to the constraints

$$\mathcal{X}_{11} + \mathcal{X}_{12} + \mathcal{X}_{13} + \mathcal{X}_{14} = 1$$
$$\mathcal{X}_{21} + \mathcal{X}_{22} + \mathcal{X}_{23} + \mathcal{X}_{24} = 1$$
$$\mathcal{X}_{31} + \mathcal{X}_{32} + \mathcal{X}_{33} + \mathcal{X}_{34} = 1$$
$$\mathcal{X}_{41} + \mathcal{X}_{42} + \mathcal{X}_{43} + \mathcal{X}_{44} = 1$$
$$\mathcal{X}_{11} + \mathcal{X}_{21} + \mathcal{X}_{31} + \mathcal{X}_{41} = 1$$
$$\mathcal{X}_{12} + \mathcal{X}_{22} + \mathcal{X}_{32} + \mathcal{X}_{42} = 1$$
$$\mathcal{X}_{13} + \mathcal{X}_{23} + \mathcal{X}_{33} + \mathcal{X}_{43} = 1$$
$$\mathcal{X}_{14} + \mathcal{X}_{24} + \mathcal{X}_{34} + \mathcal{X}_{44} = 1$$
$$\mathcal{X}_{ij} \in \{0,1\}$$

3.1. Solution Procedure

Step 1:
At the beginning, no crop is allotted to any fields; the task (σ) at the top most node is 0; the branching tree's assignment is a null set { ϕ }; and the lower level that follows is likewise 0

Step 2:
Divide it into various sub-problems stemming from the root node, ensuring a lower bound for each.
Calculate the lower bound $\mathcal{P}_{11}^{1}$

$$\mathcal{V}_{\sigma} = \sum_{i,j\in\mathcal{A}} \mathcal{C}_{i,j} + \sum_{i\in\mathcal{X}} \Big(\sum_{j\in\mathcal{Y}} min\,\mathcal{C}_{i,j} \Big)$$

Where $i, j \in \{1,1\}$, $\mathcal{A} = \{1,1\}$ $\mathcal{X} = \{2,3,4\}$, $\mathcal{Y} = \{2,3,4\}$

$$\mathcal{V}_{11} = \mathcal{C}_{11} + \sum_{i\in(2,3,4)} \Big(\sum_{j\in(2,3,4)} min\,\mathcal{C}_{i,j} \Big)$$

$$\mathcal{P}_{11}^{1} = \mathcal{C}_{11} + \{\min(\mathcal{C}_{22}, \mathcal{C}_{23}, \mathcal{C}_{24}) + \min(\mathcal{C}_{32}, \mathcal{C}_{33}, \mathcal{C}_{34}) + \min(\mathcal{C}_{42}, \mathcal{C}_{43}, \mathcal{C}_{44})\}$$

$$\mathcal{P}_{11}^{1} = 3.45 + 1.92 + 1.2375 + 7.5756$$

$$\mathcal{P}_{11}^{1} = 14.1831$$

Similarly, we can calculate,

$$\mathcal{P}_{12}^{1} = \mathcal{C}_{12} + \{\min(\mathcal{C}_{21}, \mathcal{C}_{23}, \mathcal{C}_{24}) + \min(\mathcal{C}_{31}, \mathcal{C}_{33}, \mathcal{C}_{34}) + \min(\mathcal{C}_{41}, \mathcal{C}_{43}, \mathcal{C}_{44})\}$$

$$\mathcal{P}_{12}^{1} = 5.3762 + 1.92 + 1.2375 + 6.435$$

$$\mathcal{P}_{12}^{1} = 14.9687$$

$$\mathcal{P}_{13}^{1} = 14.9481$$

$$\mathcal{P}_{14}^{1} = 14.5518$$

Step 3:

The terminal node with the lowest lower bound is where further branching is carried out, $\mathcal{P}_{11}^{1}$ and it is possible to carry out further branching from this node.

From the above values minimum cost is $\mathcal{P}_{11}^{1} = 14.1831$. Hence take out first row and first column.

Step 4:

Calculate $\mathcal{P}_{22}^{2}$ by using following steps.

$$\mathcal{P}_{22}^{2} = \mathcal{C}_{11} + \mathcal{C}_{22} + \{\min(\mathcal{C}_{33}, \mathcal{C}_{34}) + \min(\mathcal{C}_{43}, \mathcal{C}_{44})\}$$

$$\mathcal{P}_{22}^{2} = 3.45 + 6.1875 + 1.2375 + 7.5756$$

$$\mathcal{P}_{22}^{2} = 18.4506$$

$$\mathcal{P}_{23}^{2} = \mathcal{C}_{11} + \mathcal{C}_{23} + \{\min(\mathcal{C}_{32}, \mathcal{C}_{34}) + \min(\mathcal{C}_{42}, \mathcal{C}_{44})\}$$

$$\mathcal{P}_{23}^{2} = 3.45 + 6.45 + 5.4625 + 7.5756$$

$$\mathcal{P}_{23}^{2} = 22.9381$$

$$\mathcal{P}_{24}^{2} = 14.3587$$

From the above set of values, minimum value is $\mathcal{P}_{24}^{2} = 14.3587$. Additional branching is possible from $\mathcal{P}_{24}^{2}$. Delete second row and fourth column.

Step 5:

Similarly calculate the following values.

$$\mathcal{P}_{32}^{3} = 18.7912,\ \mathcal{P}_{33}^{3} = 15.3575$$

From this values, minimum value is $\mathcal{P}_{33}^{3} = 15.3575$. Eliminate third row and third column. Then the value of $\mathcal{P}_{42}^{4} = 15.3575$.

The process mentioned above can be shown as a tree,

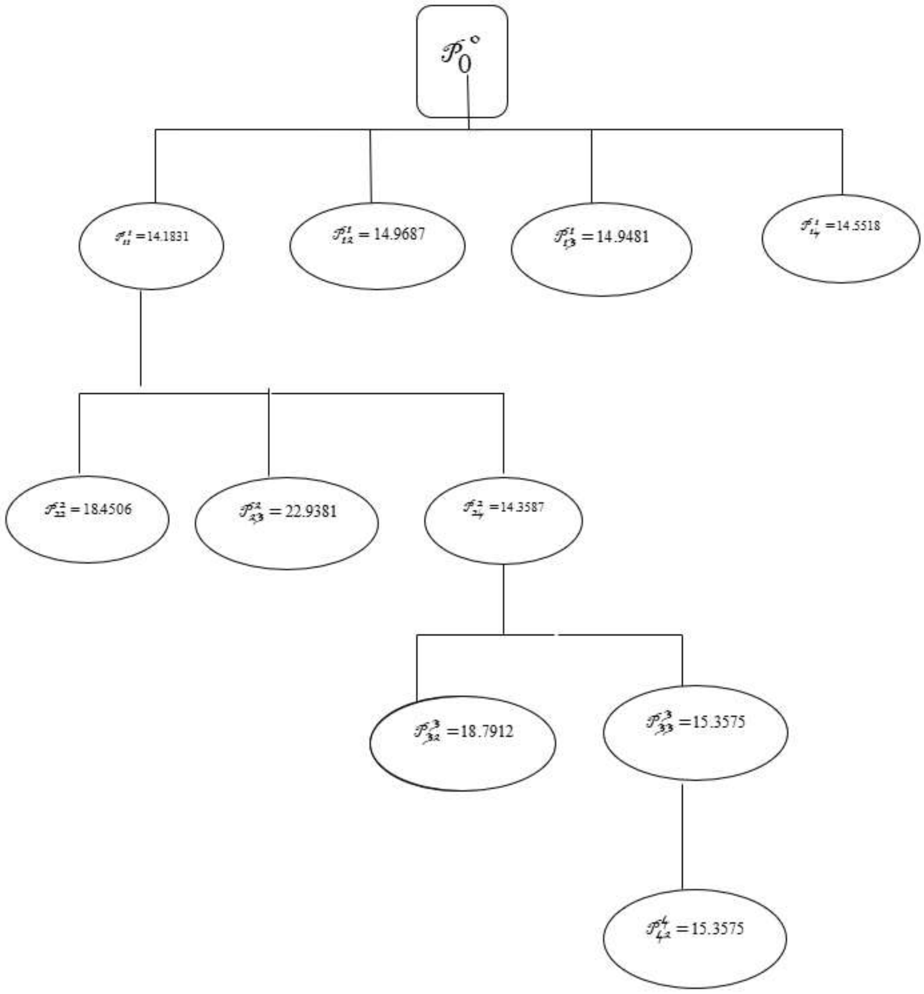


Fig 1: Branch and Bound Tree

Substitute the above values, we get the ideal allocation, (ie) **(1, 1), (2,4), (3,3), and (4,2)**. After solving, we have the optimal value is **15.3575**.

3.2. Maximization Problem

Let us examine a picture fuzzy assignment problem where the rows denote four machines and the columns signify four jobs.

	P1,	P2,	P3	P4
C1	(4,5,7,8) (0.6,0.1,0.3)	(3,6,10,12) (0.5,0.25,0.1)	(4,6,7,8) (0.4,0.3,0)	(7,8,11,12) (0.55,0.35,0.15)
C2	(2,3,4,6) (0.6,0.1,0.2)	(1,2,3,4) (0.5,0.4,0)	(9,10,112,13) (0.6,0.0.4)	(12,13,14,15) (0.5,0.1,0.2)
C3	(5,9,12,16) (0.3,0.1,0.15)	(8,11,13,16) (0.45,0.15,0.1)	(9,12,15,18) (0.5,0.4,0)	(10,11,12,13) (0.55,0.4,0)
C4	(8,9,11,15) (0.6,0.1,0.2)	(7,8,10,14) (0.7,0.2,0.1)	(2,5,7,9) (0.6,0.2,0.1)	(5,7,9,11) (0.5,0.35,0)

Determine an optimal assignment of machines to jobs that will maximize the overall profit.

Solution

By using Accuracy function we get the following cost matrix table

$$\begin{bmatrix} 3.6 & 3.7878 & 2.4062 & 5.2368 \\ 2.1937 & 1.2375 & 6.6 & 6.48 \\ 3.0318 & 4.68 & 6.6825 & 6.2818 \\ 6.2887 & 6.825 & 3.3637 & 3.91 \end{bmatrix}$$

The problem can be rewrite as follows

$$\begin{aligned} Max\ Z = {} & 3.6\mathcal{X}_{11} + 3.7878\mathcal{X}_{12} + 2.4062\mathcal{X}_{13} + 5.2368\mathcal{X}_{14} + \\ & 2.1937\mathcal{X}_{21} + 1.2375\mathcal{X}_{22} + 6.6\mathcal{X}_{23} + 6.48\mathcal{X}_{24} + \\ & 3.0318\mathcal{X}_{31} + 4.68\mathcal{X}_{32} + 6.6825\mathcal{X}_{33} + 6.2818\mathcal{X}_{34} + \\ & 6.2887\mathcal{X}_{41} + 6.825\mathcal{X}_{42} + 3.3637\mathcal{X}_{43} + 3.91\mathcal{X}_{44} \end{aligned}$$

Subject to the constraints

$$\mathcal{X}_{11} + \mathcal{X}_{12} + \mathcal{X}_{13} + \mathcal{X}_{14} = 1$$
$$\mathcal{X}_{21} + \mathcal{X}_{22} + \mathcal{X}_{23} + \mathcal{X}_{24} = 1$$
$$\mathcal{X}_{31} + \mathcal{X}_{32} + \mathcal{X}_{33} + \mathcal{X}_{34} = 1$$
$$\mathcal{X}_{41} + \mathcal{X}_{42} + \mathcal{X}_{43} + \mathcal{X}_{44} = 1$$
$$\mathcal{X}_{11} + \mathcal{X}_{21} + \mathcal{X}_{31} + \mathcal{X}_{41} = 1$$
$$\mathcal{X}_{12} + \mathcal{X}_{22} + \mathcal{X}_{32} + \mathcal{X}_{42} = 1$$
$$\mathcal{X}_{13} + \mathcal{X}_{23} + \mathcal{X}_{33} + \mathcal{X}_{43} = 1$$
$$\mathcal{X}_{14} + \mathcal{X}_{24} + \mathcal{X}_{34} + \mathcal{X}_{44} = 1$$
$$\mathcal{X}_{ij} \in \{0,1\}$$

Solution Procedure

Step 1 :
Divide it into various subproblems stemming from the root node and establish an upper limit for all.
Calculate the upper bound $\mathcal{P}_{11}^{1}$

$$\mathcal{V}_{\sigma} = \sum_{i,j\in\mathcal{A}} \mathcal{C}_{i,j} + \sum_{i\in\mathcal{X}} \Big(\sum_{j\in\mathcal{Y}} max\, \mathcal{C}_{i,j} \Big)$$

Where $i, j \in \{1,1\}$, $\mathcal{A} = \{1,1\}$ $\mathcal{X} = \{2,3,4\}$, $\mathcal{Y} = \{2,3,4\}$

$$\mathcal{V}_{11} = \mathcal{C}_{11} + \sum_{i\in(2,3,4)} \Big(\sum_{j\in(2,3,4)} max\, \mathcal{C}_{i,j} \Big)$$

$$\mathcal{P}_{11}^{1} = \mathcal{C}_{11} + \{\max(\mathcal{C}_{22}, \mathcal{C}_{23}, \mathcal{C}_{24}) + \max(\mathcal{C}_{32}, \mathcal{C}_{33}, \mathcal{C}_{34}) + \max(\mathcal{C}_{42}, \mathcal{C}_{43}, \mathcal{C}_{44})\}$$
$$\mathcal{P}_{11}^{1} = 3.6 + 6.6 + 6.6825 + 6.825$$
$$\mathcal{P}_{11}^{1} = 23.7075$$

Similarly, we can calculate,

$$\mathcal{P}_{12}^{1} = \mathcal{C}_{12} + \{\max(\mathcal{C}_{21}, \mathcal{C}_{23}, \mathcal{C}_{24}) + \max(\mathcal{C}_{31}, \mathcal{C}_{33}, \mathcal{C}_{34}) + \max(\mathcal{C}_{41}, \mathcal{C}_{43}, \mathcal{C}_{44})\}$$
$$\mathcal{P}_{12}^{1} = 3.7878 + 6.6 + 6.6825 + 6.2887$$
$$\mathcal{P}_{12}^{1} = 23.359$$
$$\mathcal{P}_{13}^{1} = 21.993$$
$$\mathcal{P}_{14}^{1} = 25.3443$$

Step 2:
The terminal node with the greatest upper bound is where further branching is carried out, $\mathcal{P}_{11}^{1}$ and it is possible to carry out further branching from this node.
From the above values maximum cost is $\mathcal{P}_{14}^{1} = 25.3443$. Hence take out first row and fourth column.

Step 3:
Calculate $\mathcal{P}_{21}^{2}$ by using following steps.

$\mathcal{P}_{21}^{2} = \mathcal{T}_{14} + \mathcal{T}_{21} + \{\max(\mathcal{T}_{32}, \mathcal{T}_{33}) + \max(\mathcal{T}_{42}, \mathcal{T}_{43})\}$

$\mathcal{P}_{21}^{2} = 5.2368 + 2.1937 + 6.6825 + 6.825$

$\mathcal{P}_{21}^{2} = 20.938$

$\mathcal{P}_{22}^{2} = \mathcal{T}_{14} + \mathcal{T}_{22} + \{\max(\mathcal{T}_{31}, \mathcal{T}_{33}) + \max(\mathcal{T}_{41}, \mathcal{T}_{43})\}$

$\mathcal{P}_{22}^{2} = 5.2368 + 1.2375 + 6.6825 + 6.2887$

$\mathcal{P}_{22}^{2} = 19.4455$

$\mathcal{P}_{23}^{2} = 23.3418$

From the above set of values, maximum value is $\mathcal{P}_{23}^{2} = 23.3418$. Additional branching is possible from $\mathcal{P}_{23}^{2}$. Delete second row and third column.

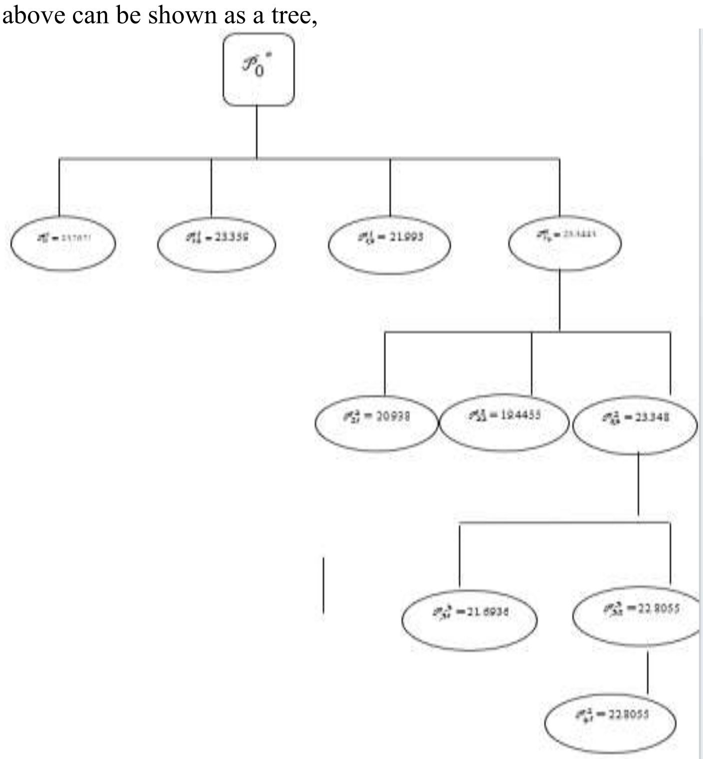
Step 4:

Similarly calculate the following values.

$\mathcal{P}_{31}^{3} = 21.6936$, $\mathcal{P}_{32}^{3} = 22.8055$

From this values, maximum value is $\mathcal{P}_{32}^{3} = 22.8055$. Eliminate third row and second column. Then the value of $\mathcal{P}_{41}^{4} = 22.8055$.

The process mentioned above can be shown as a tree,



From the above tree, the optimal allocation is **(1,4), (2,3), (3,2), (4,1)** and the optimal solution is **22.8055.**

3.3. Unbalanced Problem

Let us examine a picture fuzzy assignment problem where the rows denote three workers and the columns signify four machines.

	P1,	P2,	P3	P4
C1	(10,11,14,15) (0.5,0.4,0)	(4,6,8,10) (0.4,0.3,0)	(7,8,9,10) (0.55,0.35.0.15)	(9,10,11,12) (0.5,0.1,0.2)
C2	(5,9,12,16) (0.3,0.1,0.15)	(1,2,3,4) (0.5,0.4,0)	(4,6,8,9) (0.1,0.2,0.3)	(6,8,10,12) (0.5,0.35,0)
C3	(2,3,4,6) (0.6,0.1,0.2)	(8,11,13,16) (0.45,0.15,0.1)	(8,11,14,15) (0.5,0.1,0.2)	(5,7,9,11) (0.3,0.1,0.15)

Determine an optimal assignment of workers to machines that will minimize the overall working cost.

Solution

The given problem is unbalanced picture fuzzy assignment problem. We can solve this problem directly without using dummy row.

By using Accuracy function we get the following cost matrix table

$$\begin{bmatrix} 6.1875 & 2.695 & 4.6856 & 6.3 \\ 3.0318 & 1.2375 & 1.215 & 4.3987 \\ 2.1937 & 4.68 & 5.76 & 2.31 \\ & & & \end{bmatrix}$$

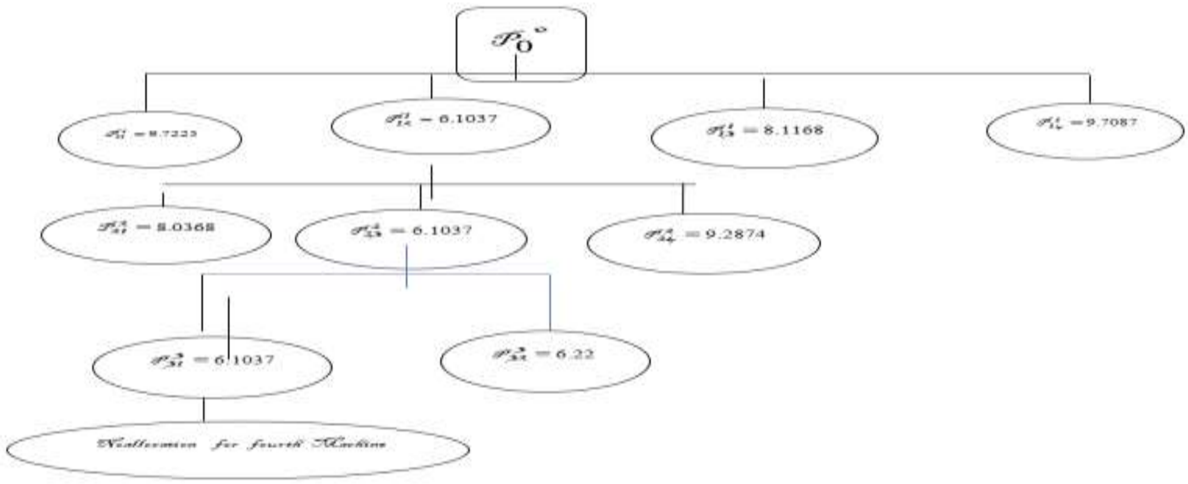
The problem can be rewrite as follows

$$\begin{aligned} Min\ Z \ = \ & 6.1875\mathscr{X}_{11} + 2.695\mathscr{X}_{12} + 4.6856\mathscr{X}_{13} + 6.3\mathscr{X}_{14} + \\ & 3.0318\mathscr{X}_{21} + 1.2375\mathscr{X}_{22} + 1.215\mathscr{X}_{23} + 4.3987\mathscr{X}_{24} + \\ & 2.1937\mathscr{X}_{31} + 4.68\mathscr{X}_{32} + 5.76\mathscr{X}_{33} + 2.31\mathscr{X}_{34} + \end{aligned}$$

Subject to the constraints

$$\mathscr{X}_{11} + \mathscr{X}_{12} + \mathscr{X}_{13} + \mathscr{X}_{14} = 1$$
$$\mathscr{X}_{21} + \mathscr{X}_{22} + \mathscr{X}_{23} + \mathscr{X}_{24} = 1$$
$$\mathscr{X}_{31} + \mathscr{X}_{32} + \mathscr{X}_{33} + \mathscr{X}_{34} = 1$$
$$\mathscr{X}_{11} + \mathscr{X}_{21} + \mathscr{X}_{31} = 1$$
$$\mathscr{X}_{12} + \mathscr{X}_{22} + \mathscr{X}_{32} = 1$$
$$\mathscr{X}_{13} + \mathscr{X}_{23} + \mathscr{X}_{33} = 1$$
$$\mathscr{X}_{14} + \mathscr{X}_{24} + \mathscr{X}_{34} = 1$$
$$\mathscr{X}_{ij} \in \{0,1\}$$

Solution Procedure
Divide it into various subproblems stemming from the root node and establish an lower limit for all.
The process mentioned above can be shown as a tree,



From the above tree, the optimal allocation is **(1,2), (2,3), (3,1)** and the optimal solution is **6.1037.**

Computational Complexity Analysis

The proposed Branch-and-Bound method systematically explores feasible assignments for the Picture Fuzzy Assignment Problem. For an n × n assignment matrix, the total number of feasible assignments is given by n!.

Therefore, in the worst case, the computational complexity of the proposed algorithm is O(n!). However, the proposed lower-bound and upper-bound strategies prune non-promising branches during the search procedure, thereby significantly reducing the computational burden in practice.

The pruning efficiency is computed a Pruning Efficiency = [(NT − NE)/NT] × 100 where NT denotes the total number of possible nodes and NE represents the number of explored nodes. Hence, the proposed method reduces unnecessary computations while preserving global optimality.

Problem Type	Problem Size	Total Assignments (NT)	Explored Nodes (NE)	Pruned Nodes	Efficiency
Minimization	4×4	24	9	15	62.5%
Maximization	4×4	24	10	14	58.33%
Unbalanced	3×4	24	7	17	70.83%

4. Discussions

The picture fuzzy assignment problem is one of the realistic problems that we have studied. When a farmer is having trouble using the limited resources at their disposal, the best way to allocate the assignment problem so that the farmer pays the least amount of money for fertilizer is as follows: The paddock P1 is assigned to crop 1**,** P2 is assigned to crop 4, P3 is assigned to crop 3, P4 is assigned to crop 2., Defuzzification is done by using accuracy function, hence the optimal value is **15.3575**. Similarly, maximization and unbalanced picture fuzzy assignment problem also solved, by using the proposed method. Hence this method is applicable for all types of assignment problem. Comparing with intuitionistic fuzzy cost [13, 16] picture fuzzy cost gives minimum optimal solution. Hence picture fuzzy numbers covering range is more than comparing with intuitionistic fuzzy numbers.

5. Conclusions

In this chapter, fertilizer expenditures are represented using **Picture Fuzzy Numbers (PFNs)** to handle uncertainty in the assignment problem. The trapezoidal picture fuzzy costs are converted into crisp values using the proposed **accuracy function**, and the optimal assignment is obtained through the **Branch-and-Bound method**. The proposed accuracy function provides better results compared with existing methods and minimizes the total fertilizer cost effectively. The computational results show high pruning efficiency, thereby reducing unnecessary computations while preserving global optimality. Hence, the proposed Picture Fuzzy Branch-and-Bound approach is both effective and computationally efficient for solving assignment problems under uncertain environments. Therefore, the proposed mathematical modelling approach serves as an effective and reliable decision-support tool for sustainable crop allocation planning in agricultural fields.

References

1. Amalendu si etal , “An approach to rank Picture fuzzy numbers for decision making problems “, Decision making applications in management and engineering, 2019, Vol 2, Issue 2, ISSN: 2560 – 6018, eISSN: 2620 – 0104, PP 54 -64. **DOI:** https://doi.org/10.31181/dmame1902049s
2. Atanassov. K. Intuitionistic fuzzy sets. Fuzzy Sets and Systems., 20,87-96(1986).
3. Chittaranjan shit, Ganeshh Ghorai etal, “Harmonic Aggregation operator with Trapezoidal picture fuzzy numbers and its application in a multiple attribute decision making problem “, Symmetry, 2022,14(1), 135, https://doi.org/10.3390/sym14010135.
4. Cuong, B. C., & Kreinovich, V.,” Picture fuzzy sets-a new concept for computational intelligence problems”, in 2013 third world congress on information and communication technologies, WICT 2013, 2013, pp. 1-6, IEEE.
5. Cuong, B. C., “Picture Fuzzy Sets-First Results, Part 1”, Seminar Neuro-Fuzzy Systems with Applications, 2013, Institute of Mathematics: Hanoi, Vietnam.
6. Cuong, B. C. “Picture fuzzy sets”, Journal of Computer Science and Cybernetics, 2014, 30(4), 409-420.
7. Das, S., Kar, S., & Pal, T., “Robust decision making using intuitionistic fuzzy numbers”, Granular Computing, 2017a, 2, 41-54.
8. Das, S., Kar, M. B., Kar, S., & Pal, T, “An approach for decision making using intuitionistic trapezoidal fuzzy soft set”, Annals of Fuzzy Mathematics and Informatics, 2018a,16(1), 99-116.
9. Fatma Kutlu Gundogdu “ Picture Fuzzy Linear Assignment Method and Its Application to Selection of Pest House Location” Intelligent and Fuzzy Techniques: Smart and Innovative Solutions, 2021, 1197, 101–109.
10. June, Muhammad akram etal, “An optimization study based on Dijkstra algorithm for a netwok with trapezoidal picture fuzzy numbers” , Neural computing and applications, 2020, DOI: https ://doi.org/10.1007/s00521 – 020 – 05034 – y.
11. Kuhn. H. W, “The Hungarian method for the Assignment Problem*”,* Noval Re.Logistics Quarterly, 1955, 83 - 97.
12. Lone. M.A etal, “Intuitionistic Fuzzy Assignment Problem: An Application in
13. Agriculture, “ Asian Journal of Agricultural Extension, Economics & Sociology, 15(4): 1-6, 2017, Article no.AJAEES.31967 ISSN: 2320-7027.

14. Li, D. F., "Extension of the LINMAP for multiattribute decision making under Atanassov's intuitionistic fuzzy environment", Fuzzy Optimization and Decision Making, 2017a, 7, 17-34.
15. Memis. S "A study on picture fuzzy sets" IFScom 2021, Turkey ISBN – 978-605- 68670-4, 2021, P. No 125 – 132.
16. Mushtaq. A.Lone , "Optimal allocation in agriculture using intuitionistic fuzzy assignment Problem", Information Science Letters, 2021-Sep, Vol 10, Issue 3, No 3, 497-501, https ://dx.doi.org/10.18576 / isl / 100314.
17. Muhammad Athar Mehmood and Shahida Bashir, "Extended Transportation Models Based on Picture Fuzzy Sets", Mathematical Problems in Engineering, Volume 2022, https://doi.org/10.1155/2022/6518976.
18. Saifullah Khan etal," Logarithmic Aggregation operators of Picture Fuzzy Numbers for multiattribute decision making problems ", Mathematics 2019, 7(7),608, https://doi.org/10.3390/math7070608
19. Sriyani violin etal, "Branch and bound algorithm analysis for solving job assignment problem", Turkish Journal of Computer and Mathematics Education, 2021 – May, Vol 12, No 11, P.No 1272 – 1276.
20. Zadeh. L. A," Fuzzy sets", Information and control, 1965, 8(3), 338 - 353.
21. Zhang, X., & Xu, Z.," A new method for ranking intuitionistic fuzzy values and its application in multi-attribute decision making", Fuzzy Optimization and Decision Making, 2012, 11, 135-146.