The Green Inventory model for reducing carbon emission using preservation technology with deteriorating items under stock dependent demand

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Abstract
In a present scenario, the inventory models are attracted by the updated technology such as Green Technology. It means incorporating the green technology into the inventory models for sustain the environment for future generations. This study is based on the inventory model for deteriorating items under stock dependent demand. The green technology is used for reducing carbon emission while manufacturing the products. The products with expiry date are need to be sold within the validity time period. The solution of this problem is to extend the validity period using preservation technology. The objective of this research paper is to reduce the carbon emission and wastages. Also maximize the profit and minimize the loss.

Keywords: Green Inventory Model, carbon emission, preservation technology, deteriorating items, stock dependent demand.

Introduction

In recent years, the concern towards environment and its sustainability are increasing. The integration of green technology into the supply chain management has become more popular among the researchers. The model aims to address the factors like controllable carbon emission, stock dependent demand, reducing deteriorating items. Green house farms are used for reducing the wastages of products, environmental impact and to meet the customer's demand. Consider a situation, a company produced thousand items for meet demand. The customers demand is decreased because of some factors. In that time green warehouse farms or preservation technology becomes more useful to rectify this problem.

Preservation technology is used for extending the validity period of the manufactured products. This technology prevents wastages and very useful while fluctuating the demand from the customers. Deteriorating items means a product that with expiry date or few days of validity period. After that period the products did not sale, the manufacturer have to destroy the products. Instead of that the technology is used for preserving the products. Using this preservation technology is reducing the wastages and minimize the loss. And while reducing wastages the environment did not affect. This model includes the optimum order quantity with own warehouse and rented warehouse, including all the costs. The ordering cost, purchasing cost, holding cost, shortage cost, lost sale cost, cyclic capital cost, transportation cost, green technology investment cost and sales revenue are calculated. The total profit and total average profit are determined for the illustrated example.

The Green technology is emerged as a critical approach to addressing environmental challenges while supporting sustainable development across various industries. It emphasizes the efficient use of resource, reduction of environmental impacts, and the integration of ecofriendly processes and materials. Preservation technology plays a significant role by extending the lifespan of products, conserving natural resources and minimizing waste generation. Previously many authors are made research in Inventory model incorporating green technology. Mashud, A. H. M., Roy, D., Daryanto, Y., & Ali, M. H. (2020) are developed a sustainable inventory model with imperfect products, deterioration, and controllable emissions [4]. Pervin, M., Roy, S. K., & Weber, G. W. (2020) are constructed Deteriorating inventory with preservation technology underprice-and stock-sensitive demand [6]. Kundu, A., Guchhait, P., Maiti, M., & Castillo, O. (2021) are formulated inventory of a deteriorating green product with preservation technology cost using a hybrid algorithm [3]. Priyamvada, Gautam, P., & Khanna, A. (2021) are developed the inventory model Sustainable production strategies for deteriorating and imperfect quality items with an investment in preservation technology [7]. Priyamvada, P., Rini, R., & Jaggi, C. K. (2022) are formulated optimal inventory strategies for deteriorating items with price-sensitive investment in preservation technology [8]. Ruidas, S., Seikh, M. R., & Nayak, P. K. (2022) are developed a production inventory model for green products with emission reduction technology investment and green subsidy [9]. Sharma, S., Tyagi, A., Verma, B. B., & Kumar, S. (2022) are created an inventory control model for deteriorating items under demand dependent production with time and stock dependent demand [10]. Sindhuja, S., & Arathi, P. (2023) are developed an inventory model for deteriorating products under preservation technology with time-dependent quality demand [11]. Chandramauli, A. B., & Santosh, S. P. (2025) are developed the model on impact of inflation on sustainable inventory model with stock dependent demand with carbon emission [1]. Pervin, M. (2025) developed a sustainable deteriorating inventory model with backorder and controllable carbon emission by using green technology [5]. Kaushik, J., (2024). An inventory model for deteriorating items towards trapezoidal type demand with preservation technology[2].

Methodology

To develop the proposed model, the following notations and assumptions are used.

Notations:

x, y, z	Constants
D	Demand
Q	Optimum order quantity
T	Order cycle replenishment time
p	Selling price
δ	Fraction per unit profit
c	Purchase cost
$I_1(t)$	Inventory model for rented warehouse
$I_2(t)$	Inventory model for owned warehouse
C_0	Capacity of owned warehouse
a, b	Deterioration rate
ρ	Capacity of owned warehouse
ε	Backorder rate
B	Backorder quantity
α	Inflation rate
β	Amount of carbon emission
PC	Purchasing cost
h_r	Holding cost for rented warehouse
h_0	Holding cost for owned warehouse
B_p	Backorder profit per unit
O_p	Ordering profit
I	Interest payment
γ	Fraction of purchasing cost
n	Number of installments
N	Trip numbers
l	Total lead time for delivery of the product
Q	Order quantity
ω	Minimum transportation profit needed to transport products
t_2	Fuel consumption when the truck is empty
d_0	Distance travelled from owned warehouse to rented warehouse and rented warehouse to customer
t_3	Supplementary of fuel consumption of the truck per ton of payload
w	Product weight
E_1	Carbon emission profit produced by the vehicle
E_2	Extra carbon emission profit for transporting one unit of an item
G_i	Green technology investment

λ Profit from using the Preservation Technology

Assumptions:

1. Planning horizon is infinite for this model.
2. Lead time of this inventory model is zero.
3. Shortages are not allowed.
4. Deterioration is happens gradually or time dependent.
5. The own warehouse deterioration is d_1 and rented warehouse deterioration is d_2.
6. The fraction of the emission reduction due to advance technology.
7. Inflation is permissible.
8. Preservation technology investment reduces the rate of deterioration with function

$$v(p,\beta) = v_0 e^{-(p-c)} \quad 0 < \beta < 1, p > 0$$

$$z(p,0) = z_{-0}$$

9. The retail price is greater than the purchasing cost per unit.
10. The fraction of emission reduction due to advanced technology G is $Y = \beta(1 - e^{fG})$.

Mathematical formulation

The proposed model consist of and rented warehouse for stocking the products. The demand is stock based and the preservation technology is used for extending the lifetime of deteriorating products. The ordering quantity of own and rented warehouse are calculated. The demand $D = (x - yp) + zI_1(t)$. All the costs and revenue are derived. Then the total profit is calculated by, subtract the total cost from the total sales revenue. Total average profit also determined. The derived model is examined by an example. In the interval $(0, t_1)$ the inventory level is decreases the products due to demand and deterioration rate.

At any time t, the inventory level $I_1(t)$ in the rented warehouse is defined by the resulting differential equation.

$$\frac{dI_1(t)}{dt} + \beta I_1(t) = -D + cI_1(t), \quad 0 \le t \le t_1$$

$$\frac{dI_1(t)}{dt} + (\beta - c)I_1(t) = -D \quad \text{.....(1)}$$

$$I_1(t) = S - c_0, \quad I_1(t) = 0 \quad \text{......(2)}$$

$$I_1(t) = \frac{D}{y-c}\left(e^{(y-c)(t_1-t_2)} - 1\right) \quad \text{.....(3)}$$

$$S = c_0 + \frac{D}{y-c}\left(e^{(y-c)t_1} - 1\right) \quad \text{....(4)}$$

Again, the level of inventory denoted by $I_0(t)$ is owned at any time t, the differential equations are,

$$\frac{dI_2(t)}{dt} + \delta I_2(t) = 0, \quad 0 \le t \le t_1 \quad \text{.....(5)}$$

$$\frac{dI_2(t)}{dt} + \delta I_2(t) = -D + cI_2(t), \quad t_1 \le t \le t_0 \quad \text{.....(6)}$$

$$\frac{dI_2(t)}{dt} = -\mu D, \quad t_0 \le t \le T \quad \text{.....(7)}$$

With $I_2(t) = c_0$, $I_2(t_0) = 0$, $I_2(T) = -B$ ......(8)

After solving the equation (5), (6), (7)

$$I_2(t) = \rho e^{-at} \quad 0 \le t \le t_1$$

$$I_2(t) = \frac{D}{a-c}\left(e^{(a-c)(t_0-t_1)} - 1\right) \quad t_1 \le t \le t_0$$

$$I_2(t) = \varepsilon D(T-t) - B \qquad t_0 \le t \le T$$

$$B = \varepsilon D(T-t_0) \qquad \text{…(9)}$$

The total order quantity is,

$$Q = S + B$$

$$Q = \rho + \frac{D}{y-c}\left(e^{(y-c)t_1} - 1\right) + \varepsilon D(T-t_0) \qquad \text{….(10)}$$

Ordering cost = O_r

Purchasing cost = $PC(S+B)$

$$= PC\left[c_0 + \frac{D}{y-c}\left(e^{(y-c)t_1} - 1\right) + \varepsilon D(T-t_0)\right] \qquad \text{…..(11)}$$

Holding Cost = $h_r \int_0^{t_1} e^{-\alpha t} I_1(t)dt + h_0 \int_0^{t_1} e^{-\alpha t} I_2(t)dt + \int_{t_1}^{t_0} e^{-\alpha t} I_i(t)dt$

$$= \frac{h_r D}{b-c}\left[e^{-\alpha t_1}\left(\frac{1}{c-b-\alpha} + \frac{1}{\alpha}\right) - \frac{e^{t_1(b-c)}}{c-b-\alpha} - \frac{1}{\alpha}\right] + h_0\rho\left[\frac{1}{\alpha+a}\left(1 - e^{-(\alpha+a)t_1}\right)\right]$$

$$+ \frac{h_0 D}{a-c}\left[\frac{e^{-\alpha t_0}}{c-\alpha-a} + \frac{e^{-\alpha t_0}}{\alpha} - \frac{e^{-\alpha t_1 + (a-c)(t_0-t_1)}}{c-\alpha-a} + \frac{e^{-\alpha t_1}}{\alpha}\right] \qquad \text{…..(12)}$$

Sales revenue $= P\left[Dt_0 + \varepsilon(T-t_0)\right]$ ….(13)

Shortage cost $= -B_p \int_{t_0}^{T} e^{-\alpha t} I_i(t)dt$ …..(14)

$$= -B_p\left[\frac{\varepsilon DT}{\alpha}\left(e^{-\alpha T} - e^{-\alpha t_0}\right) + \varepsilon D\left(\frac{t_3 e^{-\alpha t_0}}{\alpha} - \frac{Te^{-\alpha T}}{\alpha} - \frac{e^{-\alpha T}}{\alpha} + \frac{e^{-\alpha t_0}}{\alpha}\right) + \frac{B}{\alpha}\left(e^{\alpha t_0} - e^{-\alpha T}\right)\right] \qquad \text{…..(15)}$$

Lost sale cost $= C_i \int_{t_0}^{T} (1-\varepsilon) D dt$

$$= C_i(1-\varepsilon)D(T-t_0) \qquad \text{…..(16)}$$

Cyclic capital cost

The cyclic capital profits calculated using the same procedure as for the buyer receiving the order.

$$C_c = \left(\frac{I\gamma(PC)}{n} Q \times n \times \frac{l}{n}\right) + \left(\frac{I\gamma(PC)}{n} Q \times (n-1) \times \frac{l}{n}\right) + \text{.......}$$

$$\left(\frac{I\gamma(PC)}{n} Q \times \left(n - n(n-2)\right) \times \frac{l}{n}\right) + \left(\frac{I\gamma(PC)}{n} Q \times \left(n-(n-1)\right) \times \frac{l}{n}\right)$$

$$C_c = \left(\frac{I\gamma(PC)}{n} Q \times n \times \frac{l}{n}\right)\left[n + (n-1) + \text{.......} + 2 + 1\right] \qquad \text{…..(17)}$$

Transportation cost

The cost of transportation is fixed and variable cost, with the carbon emission cost. The cost of transportation is calculated for products transported from

(i) Owned warehouse to rented warehouse and

(ii) Rented warehouse to customer are paid by retailers.

Here 2d is added because distance is counted for up and down trips.

$$TC = \frac{N}{T}\left[w + \left(2dt_u t_2 + dt_u t_3 wQ\right) + 2dE_1 + dE_2 Q\right] \quad .....(18)$$

Preservation Technology cost

The cost of preservation technology refers to the expenditure that spent for extending the validity period of the manufactured products.

$$PT = \frac{(p-c)\delta Q}{T}$$

$$PT = \frac{(p-c)\delta}{T}\left[\rho + \frac{D}{y-c}\left(e^{(y-c)t_1} - 1\right) + \varepsilon D(T - t_0)\right] \quad ......(19)$$

Green Technology investment profit (GT)

Incorporating Green Technology, the reduced transportation cost is expressed as follows:

$$TC = \frac{N}{T}\left[t_1 + \left(2dt_u t_2 + dt_u t_3 wQ\right) + \left(2dE_1 + dE_2 Q\right)\left(1 - \beta\left(1 - e^{fG}\right)\right) + \lambda\right] \quad .....(20)$$

The total profits (T.P) is calculated by adding all the costs and subtracting the costs from the sales revenue.

Total profit= (Sales revenue + Green investment profit)-(all costs)

$$TP = \left\{P\left[Dt_0 + \varepsilon(T - t_0)\right] + \frac{N}{T}\left[t_1 + \left(2dt_u t_2 + dt_u t_3 wQ\right) + \left(2dE_1 + dE_2 Q\right)\left(1 - \beta\left(1 - e^{fG}\right)\right)\right]\right\}$$

$$- \left\{ \begin{array}{l} O_r + PC\left[c_0 + \frac{D}{y-c}\left(e^{(y-c)t_1} - 1\right) + \varepsilon D(T - t_0)\right] + \frac{h_r D}{b-c}\left[e^{-\alpha t_1}\left(\frac{1}{c-b-\alpha} + \frac{1}{\alpha}\right) - \frac{e^{t_1(b-c)}}{c-b-\alpha} - \frac{1}{\alpha}\right] \\ + h_0 \rho\left[\frac{1}{\alpha + a}\left(1 - e^{-(\alpha+a)t_1}\right)\right] + \frac{h_0 D}{a-c}\left[\frac{e^{-\alpha t_0}}{c-\alpha-a} + \frac{e^{-\alpha t_0}}{\alpha} - \frac{e^{-\alpha t_1 + (a-c)(t_0 - t_1)}}{c-\alpha-a} + \frac{e^{-\alpha t_1}}{\alpha}\right] \\ - B_p\left[\frac{\varepsilon DT}{\alpha}\left(e^{-\alpha T} - e^{-\alpha t_0}\right) + \varepsilon D\left(\frac{t_3 e^{-\alpha t_0}}{\alpha} - \frac{Te^{-\alpha T}}{\alpha} - \frac{e^{-\alpha T}}{\alpha} + \frac{e^{-\alpha t_0}}{\alpha}\right) + \frac{B}{\alpha}\left(e^{\alpha t_0} - e^{-\alpha T}\right)\right] + C_i(1-\varepsilon)D(T - t_0) \\ + \left(\frac{I\gamma(PC)}{n} Q \times n \times \frac{l}{n}\right)\left[n + (n-1) + ... + 2 + 1\right] + \frac{N}{T}\left[t_1 + \left(2dt_u t_2 + dt_u t_3 wQ\right) + 2dE_1 + dE_2 Q\right] \end{array} \right\}$$

.....(21)

Optimality:

(i) Find the first order derivative with respect to t_1, T, G. That is $\frac{\partial TP}{\partial t_1}, \frac{\partial TP}{\partial T}, \frac{\partial TP}{\partial G}$

(ii) Solve $\frac{\partial TP}{\partial t_1} = 0, \frac{\partial TP}{\partial T} = 0, \frac{\partial TP}{\partial G} = 0$ and find t_1, T, G.

(iii) Also find $\frac{\partial^2 TP}{\partial t_1^2}, \frac{\partial^2 TP}{\partial T^2}, \frac{\partial^2 TP}{\partial G^2}, \frac{\partial^2 TP}{\partial t_1 \partial T}, \frac{\partial^2 TP}{\partial T \partial G}, \frac{\partial^2 TP}{\partial t_1 \partial G}, \frac{\partial^2 TP}{\partial T \partial t_1}, \frac{\partial^2 TP}{\partial G \partial t_1}, \frac{\partial^2 TP}{\partial G \partial T}$.

(iv) Solving the second order derivatives, get negative value. The optimal values are consider from (ii).

Numerical example

Consider the following parameters to illustrate the proposed model:

D	\$5000
p	10
δ	0.01
c	0.1
C_0	10
a	0.01
b	0.01

ρ	\$0.1
ε	10
B	50
α	1
β	40
PC	5 lakhs
h_r	500 rupees
h_0	\$0.1
B_p	\$0.02
O_p	10 lakhs
I	100
γ	0.01
n	0.005
N	10
l	0.05
t_1	0.04
t_2	1
d_0	3900km
t_3	10
w	10
E_1	0.08
E_2	5
t_u	0.1
ε	0.001
f	100
t_0	59

Substitute the above values in the respective equations get the value of replenishment cycle time and total profit amount. And also get the green investment profit.

Results

The replenishment cycle time T^* is 335.894 days, the maximum level of inventory in t_1 is 34 days, preservation technology cost λ \$799, green investment profit \$269.117 and the total profit \$1251.039.

Discussion

The numerical analysis demonstrates the effectiveness of incorporating preservation technology into the proposed green inventory model with deteriorating items under stock-dependent demand. The optimal replenishment cycle time is 335.894 days, indicating a stable and efficient inventory policy. The variation in cycle time is negligible, suggesting that preservation technology does not significantly affect the replenishment schedule. The preservation technology incurs an additional cost of \$799, which represents the investment required to reduce the deterioration rate. The model also yields a green investment profit of \$269.117, reflecting the economic benefit of environmentally sustainable practices. The overall total profit is \$1251.039. If compared with the conventional inventory model, the total profit is reduced due to the additional preservation technology cost. However, the investment effectively minimizes deterioration losses, improves product quality, and contributes to lower carbon emissions. Although the short-term profitability decreases, the model provides significant environmental and operational benefits. These findings indicate that preservation technology supports sustainable inventory management and can be a valuable strategy for organizations seeking to balance economic performance with environmental responsibility.

Conclusion

In this paper, formulated a green inventory model for deteriorating items that incorporates preservation technology to reduce deterioration and environmental impact. The results are express that investing the preservation technology can significantly reducing the rate of deterioration and wastages of products. The total cost is minimized while maintaining product quality over the planning horizon. The model provides useful managerial insights for decision maker seeking to balance economic efficiency with sustainability goals and it offers a practical framework designing environmentally responsible inventory policies in modern supply chain.

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