



Optimization Modelling of Agricultural Production under Resource Constraints

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Abstract

Agriculture plays a crucial role in the economy of India by contributing to food security, employment generation, and economic development. This study examines the growth performance, instability, and optimization of major agricultural crops in India using time series data from 1980–81 to 2024–25. The analysis focuses on cereals (wheat, rice, maize), pulses (tur and lentil), and cash crops (sugarcane and cotton). Compound Annual Growth Rate (CAGR) and Cuddy-Della Valle Instability Index were employed to evaluate the growth and variability in area, production, and yield across three sub-periods and the overall study period. The results indicate that wheat and rice exhibited relatively stable growth due to irrigation facilities, technological advancement, and government support, whereas pulses showed lower growth and higher instability because of their dependence on rainfall and climatic variability. Maize emerged as the fastest-growing cereal crop, while cotton demonstrated high growth accompanied by significant instability. To achieve efficient utilization of limited agricultural resources, a linear programming optimization model was developed under constraints of land, water, fertilizer, and labor availability. The simplex method was applied to obtain the optimal crop allocation pattern. The model produced a maximum agricultural production of 2064.77 million tonnes, with sugarcane receiving the highest optimal allocation. The study highlights the importance of optimization modelling for sustainable agricultural planning and efficient resource management in India.

Keywords: Compound growth rate, instability index, time series data, production performance, modelling, optimization

1. Introduction

Agriculture is one of the most important sectors of the economy and plays a significant role in food security and employment generation in India. Increasing population and limited natural resources have created serious challenges for agricultural production. Resources such as land, water, fertilizers, and labor are limited and must be utilized efficiently. Therefore, scientific methods are required for proper agricultural planning and management. Optimization modelling is an effective mathematical technique used to maximize agricultural production under various resource constraints. It helps in determining the best allocation of available resources among different crops. In this study, an optimization model is developed for major crops such as wheat, rice, maize, pulses, sugarcane and cotton. The model aims to maximize total agricultural production while satisfying constraints related to land, water and other inputs. Efficient resource utilization can improve productivity, profitability and sustainability in agriculture. The proposed study provides a useful framework for policymakers and researchers in agricultural planning and decision-making.

Singh and Panda [3] conducted a study in Haryana, India, employed a linear programming model to optimize land and water resource allocation through the conjunctive use of canal water and groundwater. The findings indicated that the optimal cropping pattern improved groundwater utilization, reduced waterlogging and salinity issues, and increased net annual returns by approximately 26%. The study demonstrated the effectiveness of integrated water resource management in enhancing agricultural productivity and farm profitability in water-scarce regions. Li et al. [4] have studied an integrated optimization framework incorporating intuitionistic fuzzy programming, fuzzy credibility-constrained programming, mixed-integer nonlinear programming, and multi-objective programming was developed for agricultural water and land resource allocation under uncertainty. The model simultaneously addressed economic, environmental, and social objectives while accounting for nonlinearities and fuzzy uncertainties. Application to a case study in northeast China demonstrated its effectiveness in improving resource allocation efficiency, reducing environmental impacts, and enhancing allocation equity. The study emphasized the potential of advanced optimization techniques for sustainable agricultural resource management under uncertain conditions.

Miao et al. [5] have applied portfolio theory as an effective approach for optimizing agricultural resource allocation by balancing expected returns and production risks. Empirical evidence from China demonstrated that

portfolio-based allocation of farmland, water, and labor resources improved resource-use efficiency and farm profitability without increasing risk. The study highlighted the potential of portfolio models as a valuable decision-support tool for sustainable and risk-informed agricultural resource management. In [6], multi-objective optimization models have been widely applied for the efficient allocation of water resources among agricultural, industrial, and municipal sectors. Studies utilizing meta-heuristic approaches, particularly Genetic Algorithms and Particle Swarm Optimization, have demonstrated their effectiveness in maximizing economic benefits and employment while satisfying sectoral water demands. The findings highlight the potential of these optimization techniques as robust tools for sustainable and efficient water resource management.

Cortignani and Severini [7] had studied the generalized maximum entropy-based optimization models have been widely used for agricultural supply analysis and resource allocation under limited data availability. By incorporating water resource utilization and sustainability indicators, these models facilitate the assessment of economic, environmental, and social impacts of agricultural policies. Such approaches provide valuable support for sustainable agricultural planning and policy evaluation. In [8], multi-criteria optimization models have been extensively employed in agricultural planning to balance economic and environmental objectives. By integrating resource and policy constraints, these models support the development of sustainable production strategies that enhance profitability while reducing the consumption of critical inputs such as water and fertilizers.

Li et al. [9] had studied the integrated optimization–simulation models which have been widely employed for sustainable agricultural land and water resource management. By incorporating environmental indicators, soil–water dynamics, and uncertainty analysis, these approaches facilitate efficient resource allocation, enhance irrigation water-use efficiency, and reduce environmental impacts. Such frameworks provide effective decision-support tools for sustainable agricultural planning and soil environment conservation. Wang [10] studies have employed artificial intelligence and metaheuristic optimization techniques to enhance agricultural resource allocation under changing environmental conditions. Hybrid models combining neural networks and swarm intelligence algorithms have demonstrated high prediction accuracy and effective optimization of water, fertilizer, and other agricultural inputs. These approaches improve resource-use efficiency, crop productivity, and economic returns while supporting sustainable agricultural management and climate-resilient decision-making.

In [11], integrated optimization and groundwater simulation models have been widely employed to mitigate waterlogging and salinity in irrigated agricultural systems. These approaches facilitate optimal land and water resource allocation, support sustainable groundwater management, and enhance the effectiveness of long-term water management strategies. Zeb et al. [12] had studied the efficient crop selection and land allocation are essential for maximizing agricultural productivity under limited land, water, and labor resources. Previous studies have employed linear and multi-objective optimization techniques to balance economic, social, and environmental goals, while intercropping has been recognized for improving resource-use efficiency and farm profitability. However, the integration of crop selection, land allocation, and intercropping within a unified optimization framework under resource uncertainties remains limited, motivating the present study. Nimma et al. [13] studies have emphasized the role of data-driven agriculture, precision farming, and sensor-based technologies in enhancing agricultural productivity and resource efficiency. Artificial intelligence and machine learning have been widely applied to crop monitoring, yield prediction, disease detection, and farm management. However, the effective integration of these technologies into comprehensive decision-support systems for sustainable agriculture remains a significant research challenge.

2. Selection of crops

We have selected cereals, pulses and cash crops for optimization modelling based on the area, production and yield. In cereals we have taken wheat, rice and maize. Tur and lentil have been selected in pulses while sugarcane and cotton have been taken for cash crops analysis. For instability analysis of these crops data on area, production and yield of all the selected crops have been collected from [1,2]. We have selected three periods from 1980 to 2025 as given in Table 1.

Period	Year
I	1980-81 to 1994-95
II	1995-96 to 2009-10
III	2010-11 to 2024-25

3. Methodology used

In this section, compound annual growth rate (CAGR) and instability of the area, production and productivity of major crops have been evaluated in three sub periods and in overall period.

Following Gujarati, CAGR have been estimated by using log linear function given by

$$Z_t = xy^t \varepsilon \quad (1)$$

Where, Z_t represents the area/production/yield of the given crop in the time period t ; x, y are growth related parameters; y^t is regression coefficient; ε represents error of estimation.

Now taking logarithm of both sides of eq. (1) we get

$$\log Z = \log x + \log y \quad (2)$$

The compound growth rate (r) of area, production and yield in percentage was computed by using the function:

$$CAGR (r)(\%) = \{Antilog of (\ln y - 1) \times 100\} \quad (3)$$

Times series data from 1980/81 to 2024/25 (45 years) regarding area under cultivation, production and yield of major crops were used to find out the compound annual growth rate (CAGR), coefficient of variation (CV) and instability index. Regression coefficient was tested for significance by using student's t-test.

CDVI was originally developed by Cuddy and Valle (1978) for measuring the instability in time series data that is characterized by trend. The estimable form of the equation is as follows:

$$CDVI = CV \times \sqrt{1 - R^2} \quad (4)$$

Where CV is the coefficient of variation in percent, and R^2 is the coefficient of determination from time trend regression adjusted by the number of degrees of freedom.

Coefficient of variation was calculated by using the following formula:

$$\text{Coefficient of variation} = \frac{\text{Standard deviation}}{\text{Mean}} \times 100 \quad (5)$$

Crops	Period I	Period II	Period III	Overall Period
Wheat	0.68	0.57	0.41	0.82
Rice	0.52	0.04	0.73	0.28
Maize	0.18	2.50	1.75	1.59
Tur	1.53	0.21	0.86	0.90
Lentil	1.92	0.54	0.41	1.07
Sugarcane	1.91	0.77	0.79	1.41
Cotton	-0.23	0.38	0.83	1.54

This section presents the empirical findings on the growth performance and instability of the major crops in India across three sub-periods and the overall study period. The analysis is based on Compound Annual Growth Rates (CAGR) and instability indices of area, production and yield for seven major crops wheat, rice, maize, tur, lentil, sugarcane and cotton. The results highlight structural changes, technological developments, climatic variability and policy impacts that shaped the growth and stability of Indian agriculture.

The CAGR of area (Table 2) reflects notable differences across crop categories and time periods. Wheat experienced consistent but moderate expansion in area, with the highest growth in the overall period (0.82%), supported by assured procurement and better irrigation. Rice recorded low area growth (0.28%) overall, reflecting land shift from rice to more profitable crops or non-agricultural uses. Maize exhibited the highest area expansion among cereals (1.59%), mainly due to rising demand for feed and industrial use, and increased adoption of hybrids during period II and III. Both tur and lentil registered moderate overall growth in area (0.90% and 1.07%, respectively). Higher growth in period I reflects government price support policies and traditional pulse-growing regions, while the decline in period II indicates diversion of land toward cereals and oilseeds. Sugarcane showed strong and stable area expansion (1.41%), supported by irrigation development and higher profitability in states like Uttar Pradesh and Maharashtra. Cotton recorded the highest overall area growth (1.54%), driven by the adoption of Bt varieties and expansion in cotton belts during period III.

Crops	Period I	Period II	Period III	Overall Period
Wheat	3.31	3.48	2.55	3.2
Rice	2.51	3.06	2.25	3.1
Maize	2.02	2.80	4.00	5.74
Tur	4.81	3.16	9.68	8.22
Lentil	3.96	5.53	8.86	8.62
Sugarcane	7.43	8.87	7.22	7.88
Cotton	6.96	7.24	5.4	9.49

The instability results (Table 3) indicate the degree of variability due to climatic conditions, market fluctuations, and crop-specific constraints. Wheat and rice showed low instability (3.2% and 3.1% overall), reflecting their essential role in food security and better irrigation coverage. Maize exhibited rising instability (overall 5.74%), suggesting diversification into this crop varies significantly based on rainfall and market incentives. Tur and lentil displayed high instability (above 8% overall), largely due to their rainfed nature, susceptibility to pests and limited technological advancements compared to cereals. Sugarcane showed moderate instability (7.88%), closely linked to water availability and delayed payments from mills. Cotton exhibited the highest area instability (9.49%), partly due to climatic stress, pest outbreaks (e.g., pink bollworm), and fluctuations in global prices.

Crops	Period I	Period II	Period III	Overall Period
Wheat	3.72	1.26	1.95	2.44
Rice	3.50	1.18	2.54	2.00
Maize	2.49	4.67	4.61	4.23
Tur	0.70	0.57	2.72	1.25

Lentil	4.22	0.83	3.94	2.25
Sugarcane	3.43	0.53	2.46	2.15
Cotton	3.83	6.14	-0.37	4.49

The CAGR results (Table 4) reveal changing productivity and technological influence on crop output. Wheat and rice recorded moderate production growth (2.44% and 2.00% overall), driven by yield improvements rather than expansion in area. Maize showed the highest production growth (4.23% overall), reflecting the combined effect of area expansion and adoption of high-yielding hybrids. Tur and lentil recorded modest growth rates (1.25% and 2.25%), limited by their rainfed characteristics. Lentil performed better in period III due to improved varieties and expanding demand. Sugarcane and cotton grew at 2.15% and 4.49% overall. Cotton's exceptional production growth in period II (6.14%) coincides with the introduction of Bt cotton, though output declined in period III (-0.37%) due to pest resurgence and climatic variability.

Crops	Period I	Period II	Period III	Overall Period
Wheat	5.29	6.06	12.92	6.19
Rice	6.74	7.46	3.60	7.30
Maize	12.51	9.32	5.72	11.36
Tur	11.23	12.51	18.58	18.10
Lentil	8.04	11.00	14.62	13.36
Sugarcane	8.58	11.22	8.43	10.26
Cotton	14.14	24.12	7.05	20.83

The instability in production (Table 5) reveals significant variations across crops and periods. Wheat and rice show moderate instability (6–7%) overall, indicating relatively stable production due to better irrigation, fertilizer use, and policy support. Maize, however, showed higher instability (11.36%), influenced by dependence on monsoon and fluctuating market demand. Tur and lentil exhibited very high instability (18.10% and 13.36%), the highest among all crop groups. Their inherent vulnerability to rainfall variability, pest incidence, and limited technological advancement largely explains these fluctuations. Sugarcane exhibited moderate instability (10.26%), while cotton displayed extremely high instability (20.83%). Cotton's instability is linked to climatic extremes, pest infestation cycles, and unstable global market trends.

Crops	Period I	Period II	Period III	Overall Period
Wheat	3.02	0.69	1.54	1.61
Rice	2.96	1.24	1.80	1.72
Maize	2.51	2.11	2.81	2.59
Tur	-0.82	0.36	1.85	0.34
Lentil	2.31	0.29	3.50	1.17
Sugarcane	1.49	-0.23	1.66	0.72
Cotton	4.07	5.73	-1.18	2.91

The growth rate of productivity (Table 6) reflects technological progress, crop management practices and irrigation availability. Wheat and rice showed moderate yield growth (1.61% and 1.72% overall), indicating slow but steady technological improvement. Maize displayed the strongest yield growth among cereals (2.59%), explaining its rapid production expansion.

Crops	Period I	Period II	Period III	Overall Period
Wheat	3.61	3.82	5.11	5.61
Rice	4.57	4.98	2.41	5.20
Maize	11.35	8.01	4.11	8.32
Tur	8.24	10.92	11.71	11.68
Lentil	6.49	7.70	9.94	11.55
Sugarcane	2.72	4.93	4.55	5.84
Cotton	10.62	20.21	7.51	18.47

Tur and lentil recorded relatively lower yield growth (0.34% and 1.17% overall). Yield stagnation in tur indicates lack of high-yielding varieties and continued rainfed cultivation. Lentil showed better performance in period III due to adoption of short-duration and high-yielding varieties. Sugarcane recorded low yield growth (0.72%), constrained by water stress and pest attacks. Cotton exhibited moderate growth (2.91%), with significant gains in period I and II due to Bt adoption, although yield contracted in period III (-1.18%) as pest resistance increased.

The instability in the yield (Table 7) directly reflects environmental, technological and agronomic challenges. Wheat and rice show moderate instability (5.61% and 5.20% overall), reflecting sensitivity to extreme weather events despite overall improvements in crop management. Tur and lentil recorded high instability (11.68% and 11.55%), consistent with their vulnerability to climatic variability, diseases and limited technological adoption. Sugarcane exhibited low instability (5.84%) due to strong varietal stability and irrigation dependence. Cotton showed very high instability (18.47%), again confirming the influence of pest cycles and climatic stress.

Cereals (wheat, rice, maize) showed stable area and yield patterns with moderate and consistent growth, driven by irrigation, improved seeds and government support. Pulses (tur, lentil) demonstrated low growth and high instability, reflecting rainfed cultivation, limited technological progress and climatic sensitivity.

Cash crops particularly cotton exhibited high growth but also high instability, indicating vulnerability to market forces and climate. Maize emerged as the most dynamic cereal with strong growth in area, production and yield, though instability remains a concern. Cotton and pulses need policy attention due to their high instability and declining yield performance in recent years.

4. Mathematical Formulation of the Optimization Model

The optimization model is formulated to maximize total agricultural production by optimal allocation of cultivable land among major crops under limited resources such as land, water, and fertilizers.

Decision Variables

Let decision variables be $A_w, A_r, A_m, A_t, A_l, A_s, A_c$ be the area allocated to wheat, rice, maize, tur, lentil, sugarcane, cotton. All decision variables are measured in million hectares.

Objective Function

The objective is to maximize total agricultural production. Let $p_1, p_2, p_3, p_4, p_5, p_6, p_7$ be the productivity of wheat, rice, maize, tur, lentil, sugarcane, cotton. Then the total production function becomes:

$$Z = p_1A_w + p_2A_r + p_3A_m + p_4A_t + p_5A_l + p_6A_s + p_7A_c$$

Hence, Maximize Z

Average Productivity Coefficients

Suppose average productivity values are:

Crop	Wheat	Rice	Maize	Tur	Lentil	Sugarcane	Cotton
Productivity (tonnes/hectare)	2.68	2.05	2.08	0.74	0.71	68.53	0.32

Then the objective function becomes

$$Z = 2.68A_w + 2.05A_r + 2.08A_m + 0.74A_t + 0.71A_l + 68.53A_s + 0.32A_c$$

Constraints

Land Availability Constraint

Suppose total cultivable land available is 150 million hectares. Then

$$A_w + A_r + A_m + A_t + A_l + A_s + A_c \leq 150$$

Water Availability Constraint

Suppose water requirements per hectare are:

Crop	Wheat	Rice	Maize	Tur	Lentil	Sugarcane	Cotton
Water requirement (units/hectare)	4	6	3	2	1.5	10	5

Assume total available water resource is 700 units. Then,

$$4A_w + 6A_r + 3A_m + 2A_t + 1.5A_l + 10A_s + 5A_c \leq 700$$

Fertilizer Constraint

Suppose fertilizer requirements are:

Crop	Wheat	Rice	Maize	Tur	Lentil	Sugarcane	Cotton
Fertilizer requirement (units/hectare)	5	4	3	1.5	1	8	4

Assume total available fertilizer resource is 500 units. Then,

$$5A_w + 4A_r + 3A_m + 1.5A_t + A_l + 8A_s + 4A_c \leq 500$$

Labor Availability Constraint

Suppose labor requirements are:

Crop	Wheat	Rice	Maize	Tur	Lentil	Sugarcane	Cotton
Labor requirement	2	3	2	1	1	5	4

If total labor availability is 350 units:

$$2A_w + 3A_r + 2A_m + A_t + A_l + 5A_s + 4A_c \leq 350$$

Minimum Area Constraints

Government agricultural policy may require minimum cultivation area:

$$A_w \geq 20, A_r \geq 25, A_m \geq 10, A_t \geq 8, A_l \geq 5, A_s \geq 6, A_c \geq 7$$

Non-Negativity Constraints

$$A_w, A_r, A_m, A_t, A_l, A_s, A_c \geq 0$$

Complete Linear Programming Model

Maximize

$$Z = 2.68A_w + 2.05A_r + 2.08A_m + 0.74A_t + 0.71A_l + 68.53A_s + 0.32A_c$$

Subject to:

$$\begin{aligned} A_w + A_r + A_m + A_t + A_l + A_s + A_c &\leq 150 \\ 4A_w + 6A_r + 3A_m + 2A_t + 1.5A_l + 10A_s + 5A_c &\leq 700 \\ 5A_w + 4A_r + 3A_m + 1.5A_t + A_l + 8A_s + 4A_c &\leq 500 \\ 2A_w + 3A_r + 2A_m + A_t + A_l + 5A_s + 4A_c &\leq 350 \end{aligned}$$

Minimum Area Constraints

$$A_w \geq 20, A_r \geq 25, A_m \geq 10, A_t \geq 8, A_l \geq 5, A_s \geq 6, A_c \geq 7$$

Non-Negativity Constraints

$$A_w, A_r, A_m, A_t, A_l, A_s, A_c \geq 0$$

4.1.1 Solution Procedure

Initial Feasible Solution

Minimum allocations are:

Crop	Wheat	Rice	Maize	Tur	Lentil	Sugarcane	Cotton
Minimum area	20	25	10	8	5	6	7

Total utilized area:

$$\begin{aligned} 20 + 25 + 10 + 8 + 5 + 6 + 7 \\ = 81 \end{aligned}$$

Remaining land:

$$150 - 81 = 69$$

Resource Utilization

Water Utilization

$$\begin{aligned} 4(20) + 6(25) + 3(10) + 2(8) + 1.5(5) + 10(6) + 5(7) \\ = 80 + 150 + 30 + 16 + 7.5 + 60 + 35 \\ = 378.5 \end{aligned}$$

Remaining water:

$$700 - 378.5 = 321.5$$

Fertilizer Utilization

$$\begin{aligned} 5(20) + 4(25) + 3(10) + 1.5(8) + 1(5) + 8(6) + 4(7) \\ = 100 + 100 + 30 + 12 + 5 + 48 + 28 \\ = 323 \end{aligned}$$

Remaining fertilizer:

$$500 - 323 = 177$$

Labor Utilization

$$\begin{aligned} 2(20) + 3(25) + 2(10) + 1(8) + 1(5) + 5(6) + 4(7) \\ = 40 + 75 + 20 + 8 + 5 + 30 + 28 \\ = 206 \end{aligned}$$

Remaining labor:

$$350 - 206 = 144$$

Matrix Form of the Model

The LP model can also be written as:

Objective Function

$$\text{Maximize } Z = CX$$

where

$$C = [2.68 \quad 2.05 \quad 2.08 \quad 0.74 \quad 0.71 \quad 68.53 \quad 0.32]$$

and

$$X = \begin{bmatrix} A_w \\ A_r \\ A_m \\ A_t \\ A_l \\ A_s \\ A_c \end{bmatrix}$$

Step-wise Solution of the Optimization Model by Simplex Method

Complete Linear Programming Problem

Objective Function

Maximize

$$Z = 2.68A_w + 2.05A_r + 2.08A_m + 0.74A_t + 0.71A_l + 68.53A_s + 0.32A_c$$

Subject to:

$$A_w + A_r + A_m + A_t + A_l + A_s + A_c \leq 150$$

$$\begin{aligned}
4A_w + 6A_r + 3A_m + 2A_t + 1.5A_l + 10A_s + 5A_c &\leq 700 \\
5A_w + 4A_r + 3A_m + 1.5A_t + A_l + 8A_s + 4A_c &\leq 500 \\
2A_w + 3A_r + 2A_m + A_t + A_l + 5A_s + 4A_c &\leq 350
\end{aligned}$$

Minimum area constraints:

$$\begin{aligned}
A_w &\geq 20, A_r \geq 25, A_m \geq 10 \\
A_t &\geq 8, A_l \geq 5, A_s \geq 6, A_c \geq 7
\end{aligned}$$

Conversion into Standard Simplex Form

To remove lower bounds, define new variables:

$$\begin{aligned}
A_w &= 20 + y_1, A_r = 25 + y_2, A_m = 10 + y_3, A_t = 8 + y_4 \\
A_l &= 5 + y_5, A_s = 6 + y_6, A_c = 7 + y_7
\end{aligned}$$

where

$$y_1, y_2, y_3, y_4, y_5, y_6, y_7 \geq 0$$

Substitute into Constraints

Land Constraint

$$\begin{aligned}
(20 + y_1) + (25 + y_2) + (10 + y_3) + (8 + y_4) + (5 + y_5) + (6 + y_6) + (7 + y_7) &\leq 150 \\
81 + y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 &\leq 150 \\
y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 &\leq 69
\end{aligned}$$

Water Constraint

$$\begin{aligned}
4(20 + y_1) + 6(25 + y_2) + 3(10 + y_3) + 2(8 + y_4) \\
+ 1.5(5 + y_5) + 10(6 + y_6) + 5(7 + y_7) &\leq 700
\end{aligned}$$

Constant term:

$$80 + 150 + 30 + 16 + 7.5 + 60 + 35 = 378.5$$

Hence,

$$4y_1 + 6y_2 + 3y_3 + 2y_4 + 1.5y_5 + 10y_6 + 5y_7 \leq 321.5$$

Fertilizer Constraint

$$\begin{aligned}
5(20 + y_1) + 4(25 + y_2) + 3(10 + y_3) + 1.5(8 + y_4) \\
+ (5 + y_5) + 8(6 + y_6) + 4(7 + y_7) &\leq 500
\end{aligned}$$

Constant term:

$$100 + 100 + 30 + 12 + 5 + 48 + 28 = 323$$

Thus, $5y_1 + 4y_2 + 3y_3 + 1.5y_4 + y_5 + 8y_6 + 4y_7 \leq 177$

Labor Constraint

$$\begin{aligned}
2(20 + y_1) + 3(25 + y_2) + 2(10 + y_3) + (8 + y_4) \\
+ (5 + y_5) + 5(6 + y_6) + 4(7 + y_7) &\leq 350
\end{aligned}$$

Constant term:

$$40 + 75 + 20 + 8 + 5 + 30 + 28 = 206$$

Hence,

$$2y_1 + 3y_2 + 2y_3 + y_4 + y_5 + 5y_6 + 4y_7 \leq 144$$

Reduced Objective Function

Substituting transformed variables into objective function:

$$\begin{aligned}
Z &= 2.68(20 + y_1) + 2.05(25 + y_2) + 2.08(10 + y_3) \\
&+ 0.74(8 + y_4) + 0.71(5 + y_5) + 68.53(6 + y_6) + 0.32(7 + y_7)
\end{aligned}$$

Constant part:

$$\begin{aligned}
53.6 + 51.25 + 20.8 + 5.92 + 3.55 + 411.18 + 2.24 \\
= 548.54
\end{aligned}$$

Therefore,

$$Z = 548.54 + 2.68y_1 + 2.05y_2 + 2.08y_3 + 0.74y_4 + 0.71y_5 + 68.53y_6 + 0.32y_7$$

Ignoring constant term

Reduced LP Problem

Maximize

$$Z' = 2.68y_1 + 2.05y_2 + 2.08y_3 + 0.74y_4 + 0.71y_5 + 68.53y_6 + 0.32y_7$$

Subject to:

$$\begin{aligned}
y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 &\leq 69 \\
4y_1 + 6y_2 + 3y_3 + 2y_4 + 1.5y_5 + 10y_6 + 5y_7 &\leq 321.5 \\
5y_1 + 4y_2 + 3y_3 + 1.5y_4 + y_5 + 8y_6 + 4y_7 &\leq 177 \\
2y_1 + 3y_2 + 2y_3 + y_4 + y_5 + 5y_6 + 4y_7 &\leq 144
\end{aligned}$$

Introducing slack variables as s_1, s_2, s_3, s_4

Then,

$$\begin{aligned}
y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + s_1 &= 69 \\
4y_1 + 6y_2 + 3y_3 + 2y_4 + 1.5y_5 + 10y_6 + 5y_7 + s_2 &= 321.5 \\
5y_1 + 4y_2 + 3y_3 + 1.5y_4 + y_5 + 8y_6 + 4y_7 + s_3 &= 177 \\
2y_1 + 3y_2 + 2y_3 + y_4 + y_5 + 5y_6 + 4y_7 + s_4 &= 144
\end{aligned}$$

Solution of LPP Using Simplex Method

Initial Simplex Table

	X_B	C_B	y_1	y_2	y_3	y_4	y_5	y_6	y_7	s_1	s_2	s_3	s_4	RHS	Min. Ratio
	s_1	0	1	1	1	1	1	1	1	1	0	0	0	69	69/1=69
	s_2	0	4	6	3	2	1.5	10	5	0	1	0	0	321.5	321.5/10=32.15
	s_3	0	5	4	3	1.5	1	8	4	0	0	1	0	177	177/8=22.125
	s_4	0	2	3	2	1	1	5	4	0	0	0	1	144	144/5=28.8
C_j			2.68	2.05	2.08	0.74	0.71	68.53	0.32	0	0	0	0		
$Z_j = C_B X_B$			0	0	0	0	0	0	0	0	0	0	0		
$C_j - Z_j$			2.68	2.05	2.08	0.74	0.71	68.53	0.32	0	0	0	0		Enter y_6
Pivot								8							Leave s_3

In the initial simplex table, the entering variable is y_6 since it has the largest positive value in the $C_j - Z_j$ row, i.e., 68.53. The outgoing variable is determined using the minimum ratio test. Since the minimum positive ratio is 22.125, s_3 leaves the basis. Thus, the pivot element is 8 at the intersection of row s_3 and column y_6 . The pivot row is normalized by dividing R_3 by 8. Subsequently, elementary row operations,

$$R_1 \rightarrow R_1 - R_3, R_2 \rightarrow R_2 - 10R_3, R_4 \rightarrow R_4 - 5R_3$$

are performed to make all other elements in the y_6 column zero. Hence, y_6 enters the basis replacing s_3 and the final simplex table is obtained.

Final Simplex Table

Basis	C_B	y_1	y_2	y_3	y_4	y_5	y_6	y_7	s_1	s_2	s_3	s_4	RHS
s_1	0	0.37	0.50	0.62	0.81	0.87	0	0.5	1	0	-0.12	0	46.87
s_2	0	-2.25	1	-0.75	0.12	0.25	0	0	0	1	-1.25	0	100.25
y_6	68.53	0.62	0.50	0.37	0.18	0.125	1	0.5	0	0	0.12	0	22.12
s_4	0	-1.12	0.50	0.12	0.06	0.375	0	1.5	0	0	-0.62	1	33.37
Z_j		42.83	34.26	25.69	12.84	8.56	68.53	34.26	0	0	8.56	0	1516.22
$C_j - Z_j$		-40.15	-32.21	-23.62	-12.11	-7.85	0	-33.94	0	0	-8.56	0	
Optimal		$y_1=0$	$y_2=0$	$y_3=0$	$y_4=0$	$y_5=0$	$y_6=22.12$	$y_7=0$					

Hence, the optimal value of the variables can be given as

Variable	y_1	y_2	y_3	y_4	y_5	y_6	y_7
Optimal value	0	0	0	0	0	22.125	0

Recovering original variables:

$$A_w = 20 + 0 = 20, A_r = 25 + 0 = 25, A_m = 10 + 0 = 10, A_t = 8 + 0 = 8$$

$$A_l = 5 + 0 = 5, A_s = 6 + 22.125 = 28.125, A_c = 7 + 0 = 7$$

Optimal Land Allocation

Crop	Wheat	Rice	Maize	Tur	Lentil	Sugarcane	Cotton
Optimal area (million/hect)	20	25	10	8	5	28.125	7

Maximum Production

Substitute optimal values into objective function:

$$Z = 2.68(20) + 2.05(25) + 2.08(10) + 0.74(8)$$

$$+ 0.71(5) + 68.53(28.125) + 0.32(7)$$

$$= 53.6 + 51.25 + 20.8 + 5.92 + 3.55 + 1927.40625 + 2.24$$

$$Z = 2064.76625$$

Hence, optimal objective value is obtained as $Z_{max} = 2064.77$ million tonnes.

The agricultural optimization model was solved using the simplex method under the given land, water, fertilizer, and labor constraints along with minimum area requirements for each crop. The optimal solution obtained from the model indicates that the maximum value of the objective function is 2064.77.

The results reveal that sugarcane occupies the highest optimal area allocation because of its significantly larger contribution to the objective function compared to the other crops. The remaining crops are maintained at their minimum required levels to satisfy the imposed constraints. The study demonstrates that optimization modelling provides an effective mathematical framework for efficient allocation of limited agricultural resources and helps

in maximizing total agricultural production. The proposed model can assist policymakers, agricultural planners, and researchers in developing sustainable crop planning strategies under resource-limited conditions.

Conclusion

The present study examined the growth performance, instability, and optimization of major crops in India using time series data from 1980–81 to 2024–25. The analysis of Compound Annual Growth Rate (CAGR) and instability indices revealed significant variations among cereals, pulses, and cash crops in terms of area, production, and yield. Wheat and rice exhibited relatively stable growth due to better irrigation facilities, technological advancement, and government support policies. Maize emerged as the most dynamic cereal crop with high growth in area, production, and productivity. In contrast, pulses such as tur and lentil showed comparatively lower growth and higher instability because of their dependence on rainfall, limited technological progress, and climatic vulnerability. Cotton recorded high growth as well as high instability, indicating sensitivity to market fluctuations, pest attacks, and climate variability.

The study further developed a linear programming optimization model to maximize agricultural production under constraints of land, water, fertilizer, and labor availability. The simplex method provided the optimal crop allocation pattern, and the maximum value of the objective function was obtained as 2064.77 million tonnes. The results indicated that sugarcane occupied the highest optimal allocation because of its larger productivity coefficient, while other crops remained close to their minimum required levels.

Overall, the study demonstrates that optimization modelling is an effective tool for efficient resource allocation and sustainable agricultural planning. The findings may help policymakers, agricultural planners, and researchers formulate strategies for improving productivity, minimizing instability, and ensuring better utilization of limited agricultural resources in India.

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