International Journal of Aquatic Research and Environmental Studies 6(S4) 782-793 2026

Performance Evaluation Of An M/M(A,B)/C Queueing Model With Setup Time, Multiple Adaptive Vacation And Parallel Service

S. Poongkodi[1], E. Rameshkumar[2*], A. Savariyammal Caroline[3]

[1]Department of mathematics, Assistant Professor, School of Engineering and Technology, Dhanalakshmi Srinivasan University, Trichy
[2*]Department of mathematics, Professor, School of Engineering and Technology, Dhanalakshmi Srinivasan University Trichy
[3]Department of mathematics, Assistant Professor, M. Kumarasamy College of Engineering, Karur
EMAIL: poongkodisudharsan2013@gmail.com[1], rameshkumaremaths@gmail.com[2], caro.flor91@gmail.com[3]

Abstract

This paper studies an M/M(a,b)/c queueing system with identical servers, setup time, and multiple adaptive vacations. Customer arrival follows a Poisson process, and service times are exponentially distributed within the interval (a,b). When the queue size falls below the threshold level a, servers enter an adaptive vacation mode . After returning, if the queue length remains below a, the server stays idle or continues its vacation until the thresholdvalue is reached. Before starting service, each server undergoes an exponentially distributed setup period. The generating function of the queue length distribution and key performance measures are obtained. The cost model is used to analyze system performance under different traffic intensities. This model can be effectively applied in Smart Agriculture and IoT-based monitoring systems, where soil moisture sensors require adaptive scheduling, efficient resource utilization, and reduced response time for better agricultural management.

Keyword: Parallel batch service, homogeneous servers,MAV's , setup time, queue length distribution, LAC.

1. Introduction

An Optimal and economical resource utilization, both the queue length and the distribution of server content plays a significant role.The study of queueing systems with server vacations has become an important research area in queueing theory owing to its wide applicability in telecommunication networks, manufacturing systems, food processing industries, chemical plants, and various service environments. In traditional vacation models, servers temporarily suspend primary service or engage in auxiliary tasks during idle intervals. Various vacation policies, including single, multiple, and adaptive vacations, have been developed to improve system performance, enhance operational flexibility, and reduce energy consumption and operating costs.

Theadaptive vacation strategies are especially suitable for dynamic environments, where the decision to continue or terminate a vacation depends on the current system state or queue size. In practical systems like cloud computing, wireless sensor networks, and smart agriculture, servers often require a setup period before commencing service. This setup interval modelsare essential for preparatory operations such as system boot-up, sensor calibration, or resource initialization.

Thesmart agricultureto monitoring of soil and atmospheric conditions is essential for efficient irrigation and crop management. Motivated by these real-world needs, this study develops an M/M(a,b)/c queueing model with identical servers incorporating setup time and multiple adaptive vacations. The goal is to assess performance and aid in designing energy-aware service mechanisms capable of adapting to fluctuating arrival patterns.

The Internet of Things (IoT) has revolutionized in the field of agricultural automation through intelligent monitoring systems that integrate sensors, data collectors, and real-time processing modules. These systems must handle variable data flows constrained by limited bandwidth, energy, or computation resources. Queueing models serve as a valuable analytical tool to evaluate the performance characteristics. The M/M(a,b)/c model, with its multi-server configuration, enables the study of performance indices under controlled arrivals and parallel processing.

Previous researchershas largely focused on classical M/M/c models, assuming single arrivals and without setup or vacation effects. However, major attention has been given to systems combining batch-size dependent service, setup time, and adaptive vacations in a multi-server framework. This motivates the present work, which introduces a generalized M/M(a,b)/c queueing model that integrates these mechanisms. Using the technique of Supplementary Variable , the PGFhas formulated. Furthermore, a cost optimization framework is developed, and the cost model is applied to examine the trend of Long-run average cost (LAC) across varying traffic intensities.

This paper differs from earlier studies by providing an analytical and numerical exploration of variable batch homogeneous parallel queue with service in a system with setup time and MAV's. The model findings have practical relevance for smart agriculture and IoT-based resource management, where cost and energy optimization are key performance criteria. We observe that the M/M(a,b)/c queueing model with Setup time, MAV and parallel services the soil moisture sensor techniques is better than the manual method.

Several researchers have extensively studied various extensions of queueing models incorporating setup times, breakdowns, and vacation policies. Choudhury (2011) investigated a service system of two-phase with a Bernoulli vacation. Jeyakumar and Senthilnathan (2012) analyzed an $M^x/G(a,b)/1$ queue with variousparameter . Abu-Dayyeh and Madan(2003) presented a$M^x/G(a,b)/1$ models under random failures. Acharya S.K, Veronica S, and Villarreal C E (2013) derived maximum-likelihood estimators of heterogeneous servers in the M/M/c. Arumuganathan and Jeyakumar (2005) contributed to bulk-service queue models with diverse parameter settings. Jeyakumar and Rameshkumar (2019) examined an $M^x/G(a,b)/1$ queue with noninterruptible server breakdowns, controllable arrivals, MAV's and closedown times. Similarly, Rameshkumar and Jeyakumar (2016) studied a related model featuring closedown periods and arrival control in system withMAV's.

Kumar and Sharma (2013) performed in the system of M/M/c/N that accounts for the retention of impatient customers, while Rahim and Thiagarajan (2023) proposed a computational approach for analyzing the model of M/M(a,b)/1independent queues with rate of arrival with control.Rameshkumar and Poongkodi (2024**)** studied a general bulk queueing model with binomial service, setup times, and multiple adaptive vacations when the queue is empty.Senthilnathan and Jeyakumar (2013) considered an M^x /G(a,b)/1queue featuring setup with multiple vacation and closedown times, and server breakdowns unterrupted.Classical contributions by Medhi (1992, 2002) discussed single-server Poisson input queues incorporating an optional secondary service channel.Rakesh Kumar and Sapana Sharma (2019) explored the M/M/c queue with transientbehaviorretention of reneging customers, while Sudhesh and Azhagappan (2019) examined the same model with heterogeneous servers, balking, and reneging phenomena.Lavanya, Vennila, and Sankoh (2022) applied a bulk-service queue in ceramic-technology processes.Additionally, Kumar and Sharma (2013)has studied an M/M/c/N system of customer retention with reneging. Kalidass, Gnanaraj, and Gopinath (2014) carried out in repairable server with multiple vacations in a transient analysis for an M/M/1 queue .

In contrast to these existing works, the present paper proposes a generalized M/M(a,b)/c queueing model that incorporates batch-size variable service, setup time and MAV's within a homogeneous parallel-server environment.A keythat contribution of this study is the integration of these analytical results into a practical smart-agriculture application, demonstrating how the derived performance measures can guide decision-makers in optimizing total cost and energy efficiency.Furthermore, the paper highlights the trend and variability of the Long-run Average Cost (LAC) under changing traffic intensities, smoothing fluctuations and revealing the underlying cost behavior.Finally, the motivation for the model arises from real-world operational challenges observed in agricultural and service systems, which are addressed in the upcoming section.

InSection 2, the practical queueing problem with an illustrative example. Section 3 shows the equation of systems are developed. Section 4 discusses the probability distribution of the queue size, while Section 5 determines the PGF of the steady-state distribution. Section 6 presents different performance metrics of the queueing system are established. Sections 7 and 8 present an analytical illustration of the cost model and computational study. Finally, conclusions and directions for future work are provided in Section 9.

2. Practical Application:

In the proposed M / M(a,b) /c model with setup time, multiple adaptive vacations, idle time, and gated queue, these sensors act as data sources, where observations are generated in batches, conforming to the bulk Poisson arrival pattern.

Soil Moisture Sensors and Temperature-Humidity Sensors (such as DHT11 or DHT22**)** play a vital role in detecting environmental variations. Soil moisture sensors are employed to measure the volumetric water content in the soil, enabling farmers to schedule irrigation effectively and prevent both overwatering and drought stress. Temperature and humidity sensors continuously observe climatic conditions, offering insights into the microclimate around the crops. These sensors are integrated into Wireless Sensor Nodes that operate in energy-constrained environments and transmit data at irregular intervals based on the sensed values.

The collected data packets are temporarily stored in a buffer until a predefined threshold is reached (gated queue mechanism). Once the condition is met, the node initiates the data transmission process after a setup period, aligning with the concept of server setup time. If there are no data packets to send, the sensor node enters an adaptive sleep mode (vacation), reducing energy consumption and extending network lifetime. Idle periods are accounted during low activity phases. Multiple sensor nodes (c servers) can operate concurrently, to save the energy and cost. This architecture effectively maps the real-time functioning of smart agricultural systems into a structured queueing framework optimization.

3. Model Framework and Associated Equations

This study examines an M/M(a,b)/c model comprising multiple identical servers operating under a multiple adaptive vacation policy with setup time. Customer arrival rate according to a Poisson process with parameterλ, while the service rate according to exponentially with rate of μ. The system contains c homogeneous servers, each capable of serving customers in batches. If the serverfind 'a' customer in the queue, and up to 'b' customers can be served in a single batch (a≤ n≤ b). When the queue length drops below a, the 'c' serversuspend service and enter adaptive vacation mode to save energy. Upon completing a vacation, a server examines the current queue length. When n< a, the server either remains idle for some time or continues its break. This cycle repeats until the n ≥ a. The queue operates on a FCFS basis, with an unlimited buffer capacity. Using this framework, various key performance measures are derived**.**By using the cost model, understanding that of how LAC varies with changing arrival rates and system parameters.

3.1 Assumptions and Variable Notation

The subsequent notations are adopted in this paper:

X - Arrival random variable: size of the arriving size

X(z) - PGF of X.

λ - Rate of arrival.

μ **-** Rate of service.

n - The total count of customers waiting in line.

We define the probabilities as follows,

$U_{c,n}(t)$ = Pr{ cchannels are busy ; n : The total count of customers in the queue}

$T_{c,l,n}(t)$ =Pr{ c channels are busy ; l :length of the vacation ; n : The total count of customers in the queue}

$R_{c,n}(t)$= Pr{c channels are under setup work ; n : The total count of customers in the queue }

$U_{0,0}(t)$ = Pr{ No channels are busy with 0 customer}

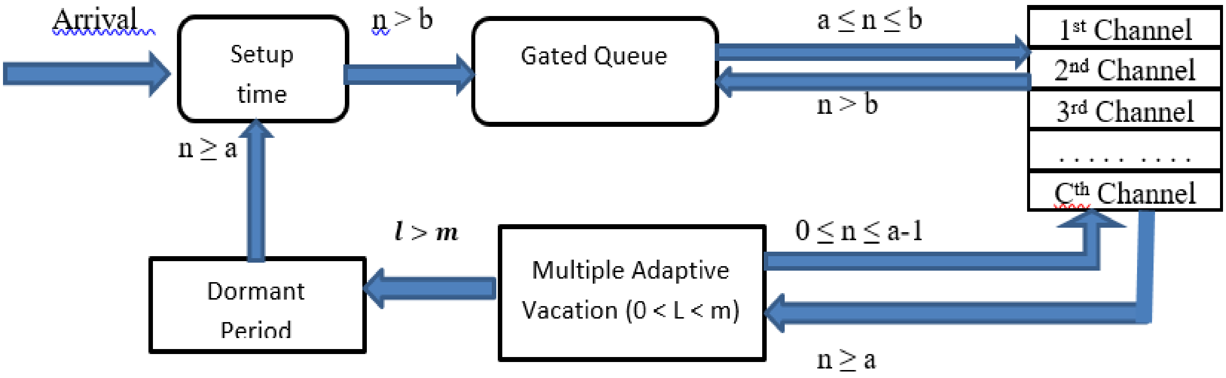

Fig. 1: Diagram illustrating the structure of the queueing model.

3.2 Mathematical Representation of Steady-State Systems

The equations for the steady-state queue size are derived as follows:

$U'_{c,n}(t) = -[\lambda + c\mu]U_{c,n}(t) + \lambda U_{c,n-1}(t) + c\mu U_{c,n+b}(t) + c\mu R_{c,n}(t), \quad n \geq a$(1)
$U'_{c,0}(t) = -[\lambda + c\mu]U_{c,0}(t) + c\mu \sum_{k=a}^{b} U_{c,k}(t) + c\mu \sum_{k=a}^{b} R_{c,k}(t)$ (2)
$U'_{c,q}(t) = -[\lambda + c\mu]U_{c,q}(t) + \lambda U_{c,q-1}(t) + c\mu \sum_{k=a}^{b} U_{c,k}(t) + \lambda \sum_{k=a}^{b} U_{c,k}(t)\, c\mu + \sum_{l=1}^{M} T_{c,l,q}(t)\, c\mu 1 \leq q \leq a-1$(3)
$U'_{0,q}(t) = -\lambda U_{0,q}(t) + \lambda U_{0,q-1}(t) + c\mu \sum_{k=a}^{b} U_{c,k}(t) + \sum_{l=1}^{M} T_{c,l,q}(t)\, \chi(x)$ (4)
$U'_{0,0}(t) = -\lambda U_{0,0}(t) + c\mu U_{c,0}(t) + R_{c,n}(0,t)\chi(x) + \sum_{l=1}^{M} T_{c,l,q}(t)\, \chi(x),$ $1 \leq n \leq a-1$(5)
$T'_{c,1,0}(t) = -[\lambda + c\mu]T_{c,1,0}(t) + U_{c,0}(t)\chi(x) + c\mu \sum_{k=a}^{b} U_{c,k}(t)$ (6)
$T'_{c,l,0}(t) = -[\lambda + c\mu]T_{c,l,0}(t) + T_{c,l-1,0}(t)\chi(x),$ $l \geq 2$ (7)
$T'_{c,1,n}(t) = -[\lambda + c\mu]T_{c,1,n}(t) + \lambda \sum_{k=1}^{n} T_{c,1,n-k}(t),$ $l \geq 2, n \geq a$(8)
$T'_{c,l,n}(t) = -[\lambda + c\mu]T_{c,l,n}(t) + \lambda \sum_{k=1}^{n} T_{c,l,n-k}(t)$ (9)
$R'_{c,n}(t) = -[\lambda + c\mu]R_{c,n}(t) + \lambda R_{c,n-1}(t) + \lambda \sum_{k=a}^{b} U_{c,k}(t)\, c\mu + c\mu R_{c,n+b}(t)$ (10)

4. Probability distribution of Queue size

The LST of $U_{c,n}(x), R_{c,n}(x), T_{c,l,n}(x)$ are defined as follows:

$$\tilde{U}(\theta)=\int_0^\infty e^{-\theta x}U_{cn}(x)dx,\ \tilde{R}_{cn}(\theta)=\int_0^\infty e^{-\theta x}R_{cn}(x)dx,\ and\ \ \tilde{T}_{cln}(\theta)=\int_0^\infty e^{-\theta x}T_{cln}(x)dx,$$

Applying the LST on both sides of the equations (1) - (10) we obtain,

$$\theta\tilde{U}_{c,n}(\theta)-U_{c,n}(0)=-(\lambda+c\mu)\tilde{U}_{c,n}(\theta)+\lambda\tilde{U}_{c,n-1}(\theta)+c\mu\tilde{U}_{c,n+b}(\theta)+c\mu\tilde{R}_{c,n}(\theta)\ ,\ n\geq a \quad (11)$$

$$\theta\tilde{U}_{c,0}(\theta)-U_{c,0}(0)=-(\lambda+c\mu)\tilde{U}_{c,0}(\theta)+c\mu\sum_{k=a}^{b}\tilde{R}_{c,k}(\theta)+c\mu\sum_{k=a}^{b}\tilde{U}_{c,k}(\theta) \quad (12)$$

$$\theta\tilde{U}_{c,q}(\theta)-U_{c,q}(0)=-(\lambda+c\mu)\tilde{U}_{c,q}(\theta)+\lambda\tilde{U}_{c,q-1}(\theta)+c\mu\sum_{k=a}^{b}\tilde{U}_{c,k}(\theta)+\lambda c\mu\sum_{k=a}^{b}\tilde{U}_{c,k}(\theta)+c\mu\sum_{l=1}^{m}\tilde{T}_{clq}(\theta),\quad 1\leq q\leq a-1 \quad (13)$$

$$\theta\tilde{U}_{0,q}(\theta)-U_{0,q}(0)=-\lambda\tilde{U}_{0,q}(\theta)+\lambda\tilde{U}_{0,q-1}(\theta)+c\mu\sum_{k=a}^{b}\tilde{U}_{c,k}(\theta)+\sum_{l=1}^{m}\tilde{T}_{clq}(0)\chi(\theta),\quad 1\leq q\leq a-1 \quad (14)$$

$$\theta\tilde{U}_{0,0}(\theta)-U_{0,0}(0)=-\lambda\tilde{U}_{0,0}(\theta)+c\mu\tilde{U}_{c,0}(\theta)+R_{c,n}(0)\chi(\theta)+\sum_{l=1}^{m}\tilde{T}_{cln}(0)\chi(\theta) \quad (15)$$

$$\theta\tilde{R}_{c,n}(\theta)-R_{c,n}(0)=-(\lambda+c\mu)\tilde{R}_{c,n}(\theta)+\lambda\tilde{R}_{c,n-1}(\theta)+\lambda c\mu\sum_{k=a}^{b}\tilde{U}_{c,k}(\theta)+c\mu\tilde{R}_{c,n+b}(\theta) \quad (16)$$

$$\theta\tilde{T}_{c,1,0}(\theta)-T_{c,1,0}(0)=-(\lambda+c\mu)\tilde{T}_{c,1,0}(\theta)+U_{c,0}(0)\chi(\theta)+c\mu\sum_{k=a}^{b}\tilde{U}_{c,k}(\theta) \quad (17)$$

$$\theta\tilde{T}_{c,1,n}(\theta)-T_{c,1,n}(0)=-(\lambda+c\mu)\tilde{T}_{c,1,n}(\theta)+\lambda\sum_{k=1}^{n}\tilde{T}_{c,1,n-k}(\theta)\ ,\quad n\geq a \quad (18)$$

$$\theta\widetilde{T}_{c,l,0}(\theta)-T_{c,l,0}(0)=-(\lambda+c\mu)\widetilde{T}_{c,l,0}(\theta)+T_{c,l-1,0}(0)\widetilde{\chi}(\theta), \qquad l\geq 2 \quad (19)$$

$$\theta\widetilde{T}_{c,l,n}(\theta)-T_{c,l,n}(0)=-(\lambda+c\mu)\widetilde{T}_{c,l,n}(\theta)+\lambda\sum_{k=1}^{n}\widetilde{T}_{c,l,n-k}(\theta) \tag{20}$$

To ascertain the distribution of queue size, we define the PGF as follows:

$$\begin{aligned}
&\widetilde{U}_c(z,\theta)=\sum_{j=0}^{\infty}\widetilde{U}_{cj}(\theta)z^j && \text{and} && \mathrm{U_c(z,0)=\sum_{j=0}^{\infty}U_{cj}(0)z^j}, \\
&\widetilde{T}_{cl}(z,\theta)=\sum_{n=0}^{\infty}\widetilde{T}_{c\ln}(\theta)z^n && \text{and} && \mathrm{T_{cl}(z,0)=\sum_{n=0}^{\infty}T_{cln}(0)z^n}, \\
&\widetilde{R}_c(z,\theta)=\sum_{k=0}^{\infty}\widetilde{R}_{ck}(\theta)z^k && \text{and} && \mathrm{R_c(z,0)=\sum_{k=0}^{\infty}R_{c0}(0)z^0}
\end{aligned} \tag{21}$$

The equations (11) & (12) are multiplied by Z^n and z^0summed over n=0 to ∞ and thenby (21) to obtain

$$\theta\widetilde{U}_c(z,\theta)-U_c(z,0)=-(\lambda+c\mu)\widetilde{U}_c(z,\theta)+\lambda z\widetilde{U}_c(z,\theta)+c\mu\left(\frac{1}{z^b}\right)\widetilde{U}_c(z,\theta)+c\mu\widetilde{R}_c(z,\theta)\ , \ n\geq a$$

$$\widetilde{U}_{cn}(z,\theta)=\frac{-c\mu\widetilde{R}_c(z,\lambda z-\lambda-\frac{c\mu}{z^b}(1-z^b))+c\mu\widetilde{R}_c(z,\theta)}{[\theta+\lambda(1-z)+\frac{c\mu}{z^b}(z^b-1)]} \tag{22}$$

$$\theta\widetilde{U}_{cn}(z,\theta)-U_c(z,0)=-(\lambda+c\mu)\widetilde{U}_c(z,\theta)+\frac{c\mu}{z^k}\widetilde{R}_c(z,\theta)+\left(\frac{c\mu}{z^k}\right)\widetilde{U}_c(z,\theta)$$

$$\widetilde{U}_{c0}(z,\theta)=\frac{-c\mu\widetilde{R}_c(z,c\mu(1-z^k)-\lambda)+c\mu\widetilde{R}_c(z,\theta)}{[\theta+\lambda+c\mu(z^k-1)]} \tag{23}$$

The above process can be extended for equations (13) & (14), we get

$$\theta\,\widetilde{U}_c(z,\theta)-U_c(z,0)=-(\lambda+c\mu)\widetilde{U}_c(z,\theta)+\lambda z^q\,\widetilde{U}_c(z,\theta)+(1+\lambda)c\mu\sum_{k=a}^{b}z^{q-k}\widetilde{U}_c(z,\theta)+c\mu\sum_{l=1}^{m}\widetilde{T}_{cl}(z,\theta), \tag{24}$$

$$\theta\widetilde{U}_0(z,\theta)-U_0(z,0)=-\lambda\widetilde{U}_0(z,\theta)+\lambda z\widetilde{U}_0(z,\theta)+c\mu\sum_{k=a}^{b}z^{q-k}\widetilde{U}_c(z,\theta)+\sum_{l=1}^{m}\widetilde{T}_{clq}(0)\widetilde{\chi}(\theta)$$

Similarly , we get

$$\widetilde{U}_{0q}(z,\theta)=\frac{c\mu\sum_{k=a}^{b}z^{q-k}[\widetilde{U}_c(z,\theta)-\widetilde{U}_c(z,\lambda z-\lambda)]+\sum_{l=1}^{m}\widetilde{T}_{clq}(\theta)(\widetilde{\chi}(\theta)-\widetilde{\chi}(\lambda z-\lambda))]}{[\theta+\lambda-\lambda z]} \tag{25}$$

The equation (15) are multiplied by appropriate power of z^0 summed over n=0 to ∞ and then (21) is used to obtain,

$$\theta\widetilde{U}_0(z,\theta)-U_0(z,0)=-\lambda\widetilde{U}_0(z,\theta)+c\mu\widetilde{U}_c(z,\theta)+R_{c,n}(0)\widetilde{\chi}(\theta)+\sum_{l=1}^{m}\widetilde{T}_{c\ln}(0)\widetilde{\chi}(\theta)$$

$$U_{00}(z,\theta)=\frac{c\mu[\widetilde{U}_c(z,\theta)-\widetilde{U}_c(z,-\lambda)]+(\sum_{l=1}^{m}\widetilde{T}_{c\ln}(\theta)+R_{cn}(\theta))(\widetilde{\chi}(\theta)-\widetilde{\chi}(-\lambda))]}{[\theta+\lambda-\lambda z]} \tag{26}$$

The equation (16) are multiplied by appropriate power of z^n summed over n=0 to ∞ and then (21) is used to obtain,

$$\theta\widetilde{R}_c(z,\theta)-R_c(z,0)=(\lambda z-\lambda)\widetilde{R}_c(z,\theta)+[\frac{c\mu}{z^b}-c\mu]\widetilde{R}_c(z,\theta)+\lambda c\mu\sum_{k=a}^{b}z^{n-k}\widetilde{U}_c(z,\theta)$$

$$\widetilde{R}_c(z,\theta)=\frac{-\lambda c\mu\sum_{k=a}^{b}z^{q-k}[\widetilde{U}_c(z,\lambda(z-1)+c\mu(\frac{1}{z^b}-1))-\widetilde{U}_c(z,\theta)]}{[\theta+\lambda(1-z)+c\mu(1-\frac{1}{z^b})]} \tag{27}$$

The equations (17) & (18) are multiplied by appropriate power of z^n and z^0 summed over n as n=0 to ∞ and then (21) is used to obtain,

$$\theta\widetilde{T}_{c,1}(z,\theta)-T_{c,1}(z,0)=-(\lambda+c\mu)\widetilde{T}_{c,1}(z,\theta)+U_{c,0}(0)\widetilde{\chi}(\theta)+c\mu\sum_{k=a}^{b}\frac{c\mu}{z^{k}}\widetilde{U}_{c}(z,\theta)$$

$$\widetilde{T}_{c,1,0}(z,\theta)=\frac{U_{c,0}(\theta)[\widetilde{\chi}(\theta)-\widetilde{\chi}(-\lambda-c\mu)]+\sum_{k=a}^{b}\frac{c\mu}{z^{k}}[\widetilde{U}_{c}(z,\theta)-\widetilde{U}_{c}(z,-\lambda-c\mu)]}{[\theta+\lambda+c\mu]} \tag{28}$$

$$\theta\widetilde{T}_{c,1}(z,\theta)-T_{c,1}(z,0)=-(\lambda+c\mu)\widetilde{T}_{c,1}(z,\theta)+\lambda\sum_{k=1}^{n}z^{k}\widetilde{T}_{c,1}(z,\theta)$$

$$\widetilde{T}_{c,l,0}(z,\theta)=0 \tag{29}$$

The equations (19) & (20) are multiplied by appropriate power of z^n and z^0 summedover n=0 to ∞ and then (21) is used to obtain,

$$\theta\widetilde{T}_{c,l}(z,\theta)-T_{c,l}(z,0)=-(\lambda+c\mu)\widetilde{T}_{c,l}(z,\theta)+T_{c,l-1}(0)\widetilde{\chi}(\theta)$$

$$\widetilde{T}_{c,1,n}(z,\theta)=\frac{T_{c,l-1}(\theta)[\widetilde{\chi}(\theta)-\widetilde{\chi}(-\lambda-c\mu)]}{[\theta+\lambda+c\mu]} \tag{30}$$

$$\theta\widetilde{T}_{c,l}(z,\theta)-T_{c,l}(z,0)=-(\lambda+c\mu)\widetilde{T}_{c,l}(z,\theta)+\lambda\sum_{k=1}^{n}z^{k}\widetilde{T}_{c,l}(z,\theta)$$

$$\widetilde{T}_{c,l,n}(z,\theta)=0 \tag{31}$$

5. The Mathematical Expression of Queue Size of PGF

5.1 PGFat various epochs

Using equations (22), (23), and (24), and set θ = 0 , the PGF of the service-completion epochcan be derived after some algebraic manipulation. Similarly, from equations (26), (28), (29), (30), (31) , the vacation epoch can be derived by setting θ = 0 and the PGF of queue size obtained and from equation (27), the size of the queue during the setup-completion phase is determined. Likewise, the PGF queue size at the idle-completion epoch follows from equations (25) and (26).

5.2 PGF of queuing size at a random point

The PGF can be expressed as follows:

$$U(z)=\widetilde{U}_{cn}(z,0)+\widetilde{U}_{c0}(z,0)+\widetilde{U}_{cq}(z,0)+\widetilde{U}_{0q}(z,0)+\widetilde{U}_{00}(z,0)+\widetilde{R}_{c}(z,0)+\widetilde{T}_{c,1,0}(z,0)+\widetilde{T}_{c,l,0}(z,0)+\widetilde{T}_{c,1,n}(z,0)+\widetilde{T}_{c,l,0}(z,0) \tag{32}$$

Using the equations (22) to (31) sub $\theta=0$ in (32), we get

$$\begin{gathered}
c\mu\lambda^{2}K_{1}K_{2}(X(z)-1)(\lambda+c\mu)T_{2}[\widetilde{R}_{c}(z,T_{1})-1]+\\
c\mu\lambda^{2}K_{1}K_{2}(X(z)-1)(\lambda+c\mu)T_{1}[\widetilde{R}_{c}(z,T_{2})-1]+\\
\sum_{l=1}^{m}c\mu[\widetilde{T}_{cl}(z,K_{1})-1]\lambda^{2}(X(z)-1)(\lambda+c\mu)T_{1}T_{2}K_{2}+\\
K_{1}K_{2}\lambda(\lambda+c\mu)T_{1}T_{2}[c\mu\sum_{k=a}^{b}z^{q-k}[\widetilde{U}_{c}(z,\lambda z-\lambda)-1]+[\widetilde{\chi}(\lambda X(z)-\lambda)-1]]+\\
\lambda K_{1}K_{2}(X(z)-1)(\lambda+c\mu)T_{1}T_{2}[c\mu[1-\widetilde{U}_{c}(z-\lambda)]+[1-\widetilde{\chi}(-\lambda)]]+\\
\lambda^{3}(X(z)-1)K_{1}(\lambda+c\mu)T_{1}T_{2}c\mu\sum_{k=a}^{b}z^{q-k}[\widetilde{U}_{c}(z,K_{2})-1]+\\
\lambda^{2}K_{1}K_{2}(X(z)-1)T_{1}T_{2}[U_{c,0}(0)[1-\widetilde{\chi}(-\lambda-c\mu)]+\sum_{k=a}^{b}\frac{c\mu}{z^{k}}[1-\widetilde{U}_{c}(z,-\lambda-c\mu)]]+\\
U(z)=\frac{\lambda^{2}K_{1}K_{2}(X(z)-1)T_{1}T_{2}T_{c,l-1}(0)[1-\widetilde{\chi}(-\lambda-c\mu)]}{\lambda^{2}(X(z)-1)(\lambda+c\mu)T_{1}T_{2}K_{1}K_{2}}
\end{gathered} \tag{33}$$

$$\begin{aligned}
&where,\\
&T_1=\lambda-\lambda X(z)-c\mu(\frac{1}{z^b}-1)\\
&T_2=c\mu(1-z^k)-\lambda\\
&K_1=\lambda(c\mu\sum_{k=a}^{b}z^{q-k}-1-z^q)+c\mu(\sum_{k=a}^{b}z^{q-k}-1)\\
&K_2=\lambda(X(z)-1)+c\mu(\frac{1}{z^b}-1)
\end{aligned}$$

5.2.1 Special Cases and Comparison with Current Research

The proposed structure can be used to create a number of established queueing models.

Particular case (i)

If c = 2 , R = 0 and U = 0 , then the equation (33) simplifies into:

$$U(z)=\frac{\lambda z^{1-a}\{[\tilde{U}_0(z,C_0)-\tilde{U}_0(z,0)]C_1C_2+[\tilde{U}_0(z,C_1)-\tilde{U}_1(z,0)]C_0C_2\}+2\mu C_1C_2[\tilde{U}_2(z,C_0)-\tilde{U}_2(z,0)]+C_0C_1[\tilde{U}_1(z,C_2)-\tilde{U}_1(z,0)]}{\lambda^2(X(z)-1)C_1C_2C_0} \tag{34}$$

Where

$$\begin{aligned}
&C_0=\lambda X(z)-\lambda\\
&C_1=\lambda X(z)-\lambda-2\mu(1+\frac{1}{z^b}+X(z))\\
&C_2=\lambda X(z)-\lambda-\mu
\end{aligned}$$

Particular Case (ii)

If c = 1 and setup time consider , then the equation (33) simplifies into:

$$U(z)=\frac{\lambda(\lambda+\mu(1-X(z))z^b\mu\left[\tilde{R}(z,0)-\tilde{R}\left(z,\lambda z-\mu(1+\left(1-\frac{1}{z^b}\right)\right)\right]-\lambda^2z^{1-a}[\lambda z^b-\lambda z^{b+1}+\mu z^b-\mu][\tilde{U}(z,\mu(z-1)-\lambda)-\lambda\tilde{U}_0(z,0)]+\mu\,(\lambda+\mu(1-z)[\tilde{U}(z,0)+\tilde{U}(z,-\lambda)\,]}{\lambda[z^b(\lambda-\lambda X(z)+\mu)-\mu](\lambda+\mu-\mu z)} \tag{35}$$

5.2.2 The equilibrium condition of the system

The condition $U(1)=1$ must be satisfied by the PGF . Use the L'Hospital rule and equate U(z) = 1 to meet the condition.

$$\begin{aligned}
&\Bigg\{cT_1E(\mu)\left(bK_1-\lambda\sum_{i=1}^{c}E(X_i)\,.E(\mu)\right)\sum_{k=a}^{b}(d_k(b-k)+cK_2E(\mu)(b-T_1\lambda.E(X).E(\mu))\sum_{k=a}^{b}(c_k(b-k)+E(\mu)\left(b-cT_2\sum_{i=1}^{c}E(X_i).E(\mu)\right)\sum_{k=a}^{b}u_0(b-k)\\
&+\left(E(\mu)\left(b-K_1\sum_{i=1}^{c}E(X_i).E(\mu)\right)+cT_2\sum_{i=1}^{n}r_i(b-\lambda K_2E(X).E(S_1))(b-c\lambda.T_1E(X).E(\mu))\right)[\lambda.E(X).E(R)+\lambda.T_2E(T).E(X)]\sum_{i=a}^{b}t_i \qquad (36)\\
&+(cK_2E(T_{cn})T_1(b-c\lambda.E(X).E(R_{cn}))+(b-K_1\lambda.E(X).E(\mu)))\sum_{i=1}^{c}E(X_i).E(\mu)\sum_{i=0}^{a-1}t_i\Bigg\}=(b-\lambda.E(X).E(\mu))(b-\lambda.E(X).E(R))(b-\lambda E(X)E(T))
\end{aligned}$$

Since c_i,p_i,q_i are the probabilities of ‘i’ customers being in the queue, it follows that left hand side of the above expression must be positive. Thus $U(1)=1$ is satisfied only iff

$$\left(z^b-\sum_{k=a}^{b}z^{q-k}(\mu(\lambda-X(z))+\tilde{\chi}(\lambda-X(z))+(c\mu(\lambda-X(z))\tilde{\chi}(\lambda-X(z))\right)>0 \quad \text{and}$$

$$\rho=((\lambda(E(X))E(\mu)+E(X))E(R))/b)-(\lambda^2.(E(X))^2E(\mu)E(T_{cnl})/b^2)$$

The above equation must be satisfied if $\rho > 1$.

Result

Equation (36) contains $b+a$ unknowns $u_0,u_1,\ldots\ldots u_{a-1},r_0,r_1,\ldots\ldots r_{a-1},t_a,t_{a+1},\ldots\ldots t_{b-1}$ and $t_{0,}t_1,\ldots\ldots t_{a-1}$. We derive a relation that expresses r_i in terms of u_i in such a way that numerator has only ‘ b ’ constants. Equation (36) becomes the PGF of the system's customer distribution involving only 'b' unknowns.Using Rouche's Theorem, $\left(z^b - c\mu(\lambda - X(z))\right)\left(z^{t-k} - \mu\tilde{\chi}(\lambda - X(z))\right)$ has $\mathrm{b}-1$ zeros inside and one on the unit circle $|z| = 1$. From complex analysis, it can be shown that this generating function possesses 'b' zeros strictly inside and on the unit circle . Since the PGF is analytic within and on the unit circle, its numerator must vanish at these zeros, leading to a system of 'b' equations. For this study, MATLAB routines were employed to compute the unknown values efficiently and accurately.

5.2.3 Result

The probability t_i can be expressed in terms of u_i as follows:

$$\pi_n = \mu \sum_{k=0}^{a-1} B_k t_{q-1} + \lambda \sum_{i=a}^{b} R_i t_{i-1}, \qquad \text{n= 0, 1, 2, 3,…, a-1}$$

Where

$$B_n = \frac{d_n + \sum_{i=1}^{n} \mu_i B_{n-i} + \sum_{k=a}^{b} h r_k t^{n-k}}{1 - \mu\beta_0 - h_0}, \quad \text{n=0,1, 2, 3,…., a-1with}$$

$$B_0 = \frac{r_0 h_0}{1 - \beta_0 - h_0}, \; d_n = \sum_{i=1}^{n} r_0 h_{n-i}$$

Also $\beta_i{}'s$ and $h_i{}'s$ are the probabilities of the ' i 'customers entering during vacation time and setup time respectively.

6. Primary Performance Indicators

6.1 Mean Length of the Busy Phase

The random variable of busy period is denoted by 'τ' . Then the projected length of busy period is

$$A(\tau) = A(A_1) / \sum_{k=0}^{a} d_k + \left(A(A_1) + A(A_2)\right) / \sum_{i=a}^{b-1} u_i t_i,$$

where, $A(A_1) = A(\mu) + A(R_{cn}), A(A_2) = cA(\mu) + A(Q_{cl})$

6.2 Mean Length of the Idle Phase

Theorem :

Let 'I_{ϖ}' be the random variable of idle period and 'I_{φ}' the Idle period due to multiple vacation is given by,

$$A(I_{\varpi}) = A(I_{\varphi}) + A(R).$$

$$A(I_{\varphi}) = \frac{A(W)}{1 - \sum_{n=a}^{b-1} T_{c\ln}(0)} = \frac{A(W)}{1 - \sum_{n=a}^{b-1} \sum_{k=a}^{b} \left\{ \sum_{j=0}^{n-1} \omega_j \varphi_k \delta_{n-i-j} \right\} u_i t_j} \tag{37}$$

Proof:

We define the Random variable Y asfollows :

$$\text{Y=} \begin{cases} 0, & \text{The server breaks the idle period if atleast 'a' customers are in the system} \\ 1, & \text{The server remains idle if n < a} \end{cases}$$

The Average length of idle time due to multiple vacations $A(I_1)$ is given by

$$A(I_\varphi) = A(I_\varphi / Y = 0) U(Y = 0) + A(I_\varphi / Y = 1) + A(W) P(Y = 0) + \left(A(W) + A(I_\varphi)\right) U(Y = 1)$$

Solving for $A(I_\varphi)$ we have, $A(I_\varphi) = A(W) / U(Y = 0).$

$T_{c\ln}(0)$ = coefficient of z^{n} in $T_{1n}(z, 0)$

$$U(Y = 0) = 1 - \sum_{n=a}^{b-1} T_{CLn}(0) = 1 - \sum_{n=a}^{b-1} \sum_{k=a}^{b} \left\{ \sum_{j=0}^{n-1} \omega_j \beta_k \delta_{n-i-j} \right\} u_i t_j$$

where ω_i, φ_i are the probabilities that ' i ' customers arrive during vacation and closedown time respectively. Now the projected idle time $A(I_\varphi)$ is obtained as

$$A(I_{\varpi}) = A(I_{\varphi}) + A(R).$$

6.3 Mean Queue Size at an Arbitrary Time Instant

The Average queue length A(Q) at an arbitrary time epoch is obtained by differentiating U(z) at z=1 and given by

$$A(Q) = \frac{\begin{array}{l}((b-E(X))^2 G_{16} + (b-(A(X))^2)^2 W_{30})\sum_{i=a}^{b-1} u_i t_k + ((b-\mu_{12})^2 J_{18} + 28(b-\mu_{21})^2 W_{12}) \sum_{n=0}^{a-1} iU_i^{(1)} \\ + (b-W_{23})^2 (b-c\mu)^2 J_1 J_2 \sum_{i=0}^{a-1} U_i t_j + (b-\mu_{11})^2 J_2 \sum_{k=a}^{b} kq_k \sum_{n=0}^{a-1} iU_i \\ + (G_{14}(b-W_{14})^2 W_{13} + (b-\mu_{21})^2 W_{13})\sum_{k=a}^{b-1} ku_k \sum_{i=0}^{a-1} t_i + ((b-\mu_{21})^2 W_{14} - (b-W_{21})^2 (b-\mu_{11})^2 G_{14})\sum_{n=0}^{a-1} it_i \\ + (b-V_{21})^2 W_{23} \sum_{i=a}^{b-1} (b-i)C_i + (b-\mu_{21})W_{26} \sum_{i=a}^{b-1} ((b-12)-k^2(i-1))C_i \\ + (b-\mu_{11})^2 W_{12} \sum_{i=a}^{b-1} U_i + (b)^2 G_{26} \sum_{i=a}^{b-1} (b(b-24) - j(j-1)^3)(C_k + U_1)\end{array}}{C_n (\lambda X_{i1})^6 (b-\mu_{24})^3 (b-\mu_{24})^6} \tag{38}$$

Where ,

$W_{11} = G_{11}\mu_{11}(R_1 + T_1) + G_{21}\mu_{11}(W_2 + R_c + 2W_1 R_{1C}) - 2G_{21}\mu_{22}(W_1 + R_{c1}), W_{12} = J_{19}\mu_{11}(W_1 + R_c)$

$W_{13} = 2G_{17}\mu_{11}R_{11}Q_1 + G_{20}\mu_{11}W_3 - 5G_{11}\mu_{12}W_1, W_{14} = G_{14}\mu_{11}W_1$

$W_{15} = G_{11}\mu_{11}R_{12} - 2G_{18}J_1,\ W_{16} = G_{11}\mu_{11}, W_{11} = (C_k \lambda X_i)(b-\mu_{12})$

$W_{20} = G_{21}J_1 W_{11} - G_{21}W_{20}, W_{26} = G_{21}\mu_{11} - W_{12}\lambda X_1, G_{21} = (c\lambda X_1)(b-\mu_{21})$

$J_1 = ((\lambda X_1)C_k - (\lambda W_2)R_1), J_2 = 2(\lambda X_i)R_c$

6.4Mean Customer Waiting period

The Average waiting time is obtained by using the Little's formula as;

$$A(W) = \frac{A(Q)}{\lambda A(X)}$$

where $A(Q)$ is given in (36).

7. Cost model

We now formulate the expression of cost function of the system under the following assumptions: Let ψ_s denote the startup cost, ψ_h refer to the per-customer holding expense, ψ_0 be the operational expense on a per time basis, ψ_r be the benefit gained on a per time basis during the vacation phase, and ψ_u denote the setup cost per unit of time period. The duration of a cycle is composed of the idle period and the busy period. The projected cycle length $A(Q_\alpha)$, can be expressed as,

$$A(Q_\alpha) = A(I_\varphi) + A(\beta) = \frac{A(W)}{P(Y=0)} + A(R) + (A(\mu_1) + E(\mu_2)) / \sum_{j=0}^{a-1} \sum_{i=a}^{b-1} u_i t_j.$$

The long-run mean cost per unit time consists the sum of the startup expense per cycle, the queue holding cost, the operational cost, and the setup charge and less the vacation reward per cycle.

Averagecost=

$$\left[\psi_s + \frac{A(R)A(\beta)}{A(Q_\alpha)} - \psi_r \frac{A(W)}{U(Y=0)} + \psi_U A(R)\right] \frac{1}{A(Q_\alpha)} + \psi_U A(Q) + \psi_r(\rho)$$

where, $\rho = \lambda^2 (A(X))^2 [A(R) + \psi A(\mu)] / \psi b.$

Hence, a straightforward numerical search procedure is employed to determine the value of 'a' that produces the minimum cost.
Step 1. Fix the maximum batch size as 'b'
Step 2. Select the value of 'a' satisfying
$TC(a-1) \langle TC(a) \langle TC(a+1)$, $2 \le a \le b-1$
Step 3. The value of 'a' that meet the above inequality is taken as the optimal threshold, since it corresponds to the long-run average cost.
Using this procedure, one can identify the value of 'a' that minimizes the overall cost function.

8. Numerical illustration

The necessary roots of the function $\left(z^{q-k} - \tilde{\chi}(\lambda - X(z))\right)\left(z^{b} - \tilde{\chi}(\lambda - X(z))\right)$ are generated by using MATLAB computational approach.

8.1Analysisof Threshold Parameters and Long-run Average Cost in Smart Agriculture using M/M(a,b)/c Queueing Model

Illustrative Example: Smart Crop Processing in Precision Agriculture

We consider a real-time application in smart agriculture , multiple IoT-enabled sensor nodes monitor field conditions and send harvesting instructions. Harvested crop batches from segmented plots arrive at a centralized processing center. These batches arrive in bulk, modeled as a Poisson process.

- When the queue contains fewer crop batches than the minimum threshold a, the servers either stay idle or switch to a reduced-power vacation state.
- Once the queue size meets or surpasses the threshold a, the system initiates the setup procedure, requiring an additional setup duration prior to commencing service.
- Each server can handle up to 'b' batches in a single operating cycle.
- The service time for each batch follows an exponential distribution.
- The system contains 'c' identical servers working in parallel.

When a server stays idle for an extended interval, its vacation duration is proportionally extended. Upon returning, the server incurs a setup time before resuming service.This model accounts the idle periods, reflecting real-time challenges in agricultural automation such as machine maintenance, low energy mode operations, or delayed batch arrivals.

8.2 Parameter Assumptions for Numerical Study:

A numerical model is analyzed with the following assumptions:

Arrival rate	10 batches/hour
Service rate	6 batches/hour
Number of servers (c)	3 (for Table 1 analysis)
Thresholds (a, b)	a = 2, b = 20
Setup cost	0.2
Cost per batch (water/energy)	₹5
Holding cost per batch	0.50

The numerical results presented in Table 1 are obtained for a fixed server configuration of c = 3.

a	A(Q)	A(B)	A(I)	A(W)	LAC
2.0	3.1838	0.4578	0.5452	0.3457	10.9806
3.0	3.9552	0.4144	0.5967	0.3526	10.2515
4.0	4.2547	0.3968	0.5748	0.3654	9.9684
5.0	4.4244	0.3854	0.6011	0.3881	7.6854
6.0	5.8853	0.3758	0.6955	0.4109	7.2508
7.0	6.2589	0.3750	0.7462	0.4273	7.1111
8.0	6.8571	0.3700	0.7986	0.4458	6.8597
9.0	7.2547	0.3645	0.7952	0.4621	6.8758
10.0	**9.6953**	**0.3699**	**0.7009**	**0.4765**	**6.5471**
11.0	9.1495	0.3010	0.6004	0.4893	6.9852
12.0	9.8541	0.3102	0.6545	0.5981	7.2547
13.0	9.1010	0.3758	0.6510	0.6541	8.2456
14.0	9.1256	0.3965	0.7521	0.7415	9.2456

Table 1: Variation of Performance Measures and Long-run Average Cost for Different Threshold Values

To study the effect of server configuration on the long-run average cost, additional experiments were carried out for different server counts while keeping the remaining parameters unchanged.

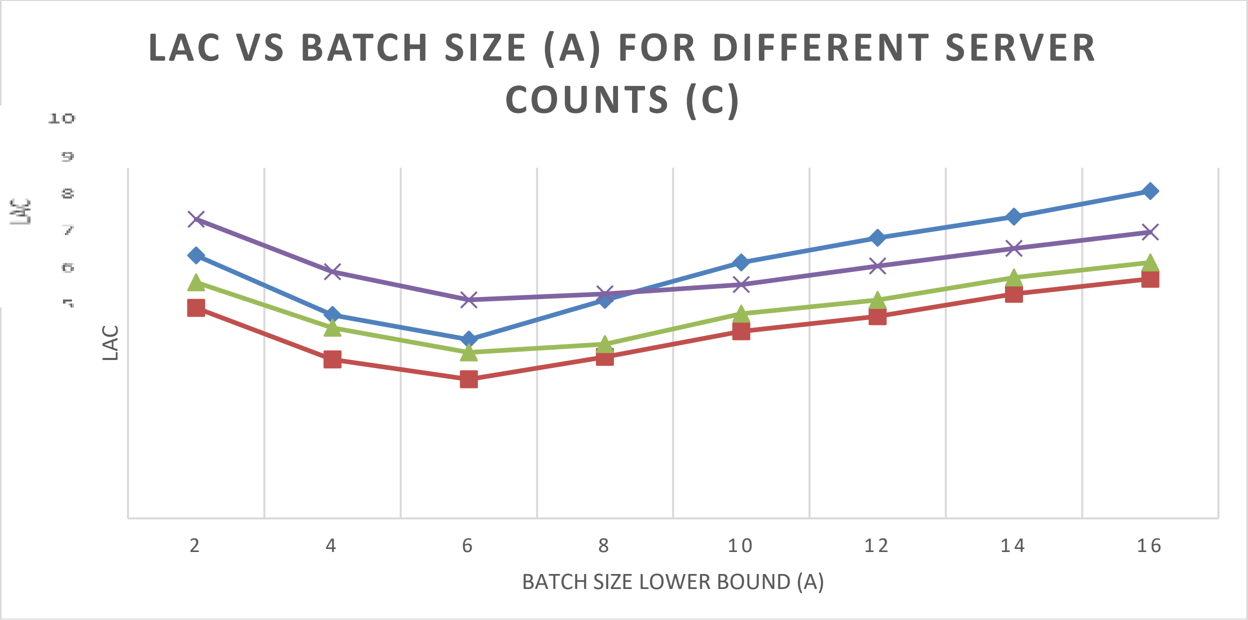

Fig. 2. Variation of Long-run Average Cost (LAC) with Batch Size (a) for Different Server Configurations (c)

In Fig. 2 illustrates the variation of the Long-run Average Cost (LAC) with respect to the threshold value a for different server configurations in the M/M(a,b)/c queueing model, with b fixed at 20. It is observed that the LAC decreases initially as the threshold value increases, reaches its minimum around a = 6, and then gradually increases for higher threshold values. Among the server configurations considered, c = 4 produces lower LAC values compared to the other configurations. These results indicate that an appropriate combination of threshold value and server count can effectively reduce the operational cost and improve system performance. Therefore, a = 6 and c = 4 can be considered an efficient configuration for minimizing the Long-run Average Cost.

8.3 Performance Metrics under Varying Threshold Values

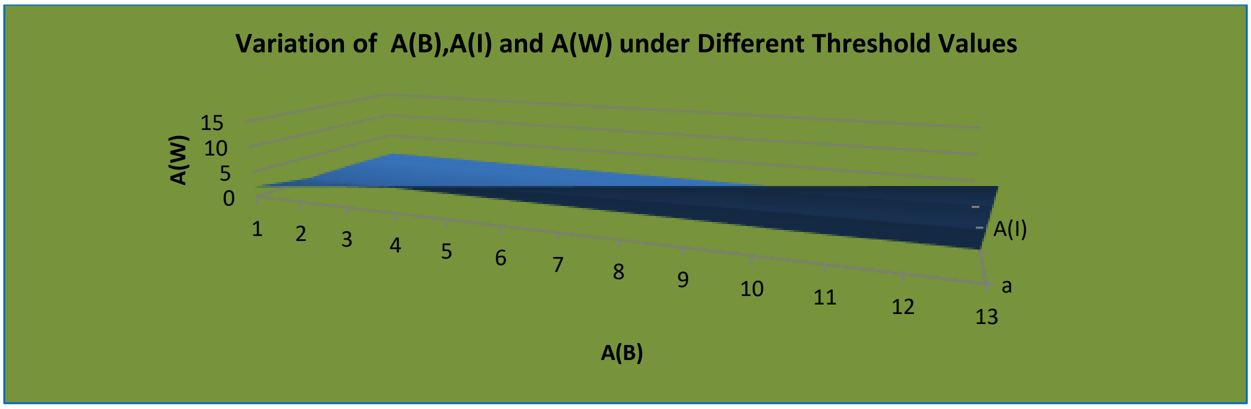

Fig. 3. Variation of A(B), A(I), and A(W) with Respect to Threshold Values

From Fig. 3, it is observed that the threshold value has a significant influence on the performance measures A(B), A(I), and A(W). As the threshold value increases, noticeable variations are observed in server utilization, idle time, and customer waiting time. In particular, A(W) exhibits an increasing trend, indicating that higher threshold values may lead to longer customer waiting times. These results demonstrate a trade-off between efficient server utilization and customer delay. Therefore, an appropriate threshold value should be selected to achieve a balance between system efficiency and acceptable customer waiting time.

8.4 Impact of Arrival Rate on Key Performance Indicators

The performance indicators such as the Average queue length A(Q), the Average duration of idle timeA(I), the Average busy time A(B), and the Average waiting timeA(W) are evaluated for different combinations of arrival rates and service rates. The outcomes of these computations are summarized in Table 2 and 3. For this analysis, the parameters are taken as a = 2, b = 5, and c = 6.

Arrival rate Vs. Performance measures for μ = 3 and for μ= 3.5

Table – 2: **Table – 3:**

λ	ρ	A(Q)	A(B)	A(I)	A(W)
0.2	0.85	0.0062	1.2500	1.6667	1.4484
0.4	0.70	0.0941	1.6667	0.8333	1.3421
0.6	0.68	0.9985	1.9852	0.6584	1.3052
0.8	0.54	1.5321	2.5000	0.5556	1.2996
1.0	0.45	2.5888	4.7560	0.4156	1.1999
1.2	0.35	5.9256	5.2306	0.3336	1.1556

λ	ρ	A(Q)	A(B)	A(I)	A(W)
0.2	0.95	0.0052	1.2554	1.6668	1.2138
0.4	0.75	0.0101	1.6565	0.8334	1.2063
0.6	0.68	0.9854	1.9852	0.5654	1.1698
0.8	0.62	1.5556	2.8562	0.6525	1.1002
1.0	0.54	2.5820	5.5262	0.5621	1.0023
1.2	0.41	4.2536	5.8561	0.3456	0.8354

From Tables 2 and 3, it is evident that increasing the service rate from μ = 3 to μ = 3.5 leads to an improvement in system performance. The expected waiting time, A(W), decreases consistently for all values of traffic intensity (ρ). For instance, at ρ = 0.2, A(W) decreases from 1.4484 to 1.2138, while at ρ = 0.6, it decreases from 1.3052 to 1.1698. In addition, noticeable changes are observed in other performance measures, such as A(Q) and A(B). These findings demonstrate that a higher service rate enhances the operational efficiency of the queueing system by reducing customer waiting time and providing better service, particularly under higher traffic conditions.

Arrival rate Vs. Performance measures for μ = 4 for μ= 4.5

Table – 4:

λ	ρ	A(Q)	A(B)	A(I)	A(W)
0.2	0.75	0.0085	0.2774	1.6008	0.8261
0.4	0.62	0.1045	1.6455	0.5623	0.8002
0.6	0.54	0.4565	1.9584	0.6235	0.7854
0.8	0.48	0.5652	2.0252	0.7245	0.6521
1.0	0.36	0.5622	4.2562	0.8725	0.5464
1.2	0.75	0.0085	5.2774	0.9008	0.5003

Table – 5:

λ	ρ	A(Q)	A(B)	A(I)	A(W)
0.2	0.95	0.1185	0.1752	1.6118	0.6856
0.4	0.85	0.1252	1.0541	0.8623	0.6052
0.6	0.74	0.9854	1.6584	0.7541	0.5221
0.8	0.65	1.4562	2.0252	0.1545	0.4654
1.0	0.47	2.7896	3.0256	0.9525	0.3564
1.2	0.21	3.6974	3.6584	0.1056	0.2564

From Tables 4 and 5, it is observed that increasing the service rate from μ = 4 to μ = 4.5 improves the overall performance of the queueing system. In particular, the expected waiting time, A(W), decreases across all traffic intensity level (ρ). The results indicate that a higher service rate enables customers to be served more efficiently, thereby reducing the average waiting time and improving system effectiveness. Therefore, increasing the service capacity enhances the operational performance of the system, especially under higher traffic conditions.

Waiting Time Analysis for Different Service Rates in M/M(a,b)/c Queueing System

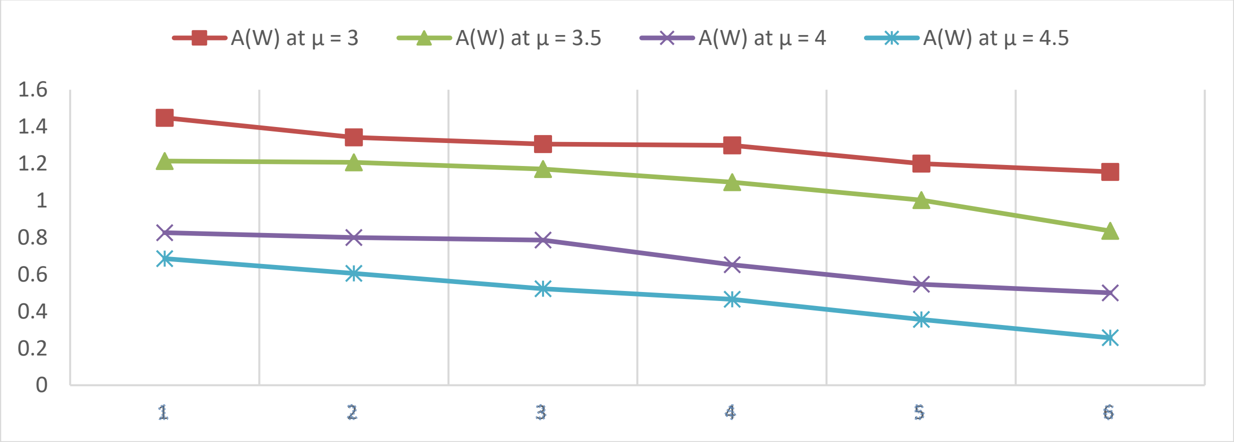

Fig.4- Variation of Expected Waiting Time A(W) for Different Service Rates

Fig. 4 illustrates the variation of the expected waiting time, A(W), for different service rates. It can be observed that, for a given arrival rate (λ), the expected waiting time decreases as the service rate (μ) increases. The curve corresponding to μ = 3 exhibits the highest waiting time, whereas the curve for μ = 4.5 shows the lowest waiting time throughout the range of arrival rates considered. These results indicate that increasing the service rate improves the efficiency of the queueing system by reducing customer waiting time. Therefore, a higher service rate contributes to better system performance, particularly under moderate and heavy traffic conditions.

9. Conclusion

This study analysed an M/M(a,b)/c queueing model incorporating setup time, multiple adaptive vacations, and gated service mechanisms. The performance of the system was evaluated in terms of the Long-run Average Cost (LAC) under different arrival rates and server configurations. Furthermore, an increase in the service rate leads to a reduction in the average waiting time, thereby improving the overall efficiency of the system. These findings can be effectively applied in smart agriculture environments, particularly in IoT-based systems where adaptive scheduling and cost management are essential. By employing such queueing models, agricultural monitoring systems can achieve improved response times and efficient resource utilisation. In addition, the numerical results indicate that the incorporation of setup time, multiple adaptive vacations (MAV), and parallel service mechanisms enhances the performance of soil moisture sensor-based monitoring systems when compared with conventional manual methods. This improvement contributes to reduced energy consumption, better irrigation management, and enhanced operational efficiency in smart agriculture applications.

References

[1] ArumuganathanR ,Jeyakumar S(2005), ‘Steady state analysis of a bulk queue with multiple vacations, setup times with N-policy and closedown times’, Applied Mathematical Modelling ,Vol.29, pp. 972 – 986. DOI: https://doi.org/10.1016/j.apm.2005.02.013

[2] Choudhury G, Tadj L, and J. C. Ke(2011), ‘A two-phase service system with bernoulli vacation schedule, setup time and **N** -policy for an unreliable server with delaying repair’, Quality Technology & Quantitative Management, Vol.8, No.3, pp. 271 – 284. DOI: https://doi.org/10.1080/16843703.2011.11673259

[3] Jeyakumar S and Senthilnathan B(2012), ‘A study on the behaviour of the server breakdown without interruption in a $M^x/G(a, b)/1$ queueing system with multiple vacations and closedowntime’, Applied Mathematics and Computation Vol.219,No.5,pp. 2618 – 2633.DOI: https://doi.org/10.1016/j.amc.2012.08.096

[4] Jeyakumar S and Rameshkumar E(2019), ‘A study on $M^x/G(a, b)/1$ Queue with server breakdown without interruption and \ controllable arrivals during multiple adaptive vacation’,International Journal of Mathematics in operational Research, Vol.15, No.2,pp.137-155.DOI: https://doi.org/10.1504/IJMOR.2019.101617

[5] Kumar R and Sharma S K(2013), ‘An M/M/c/N queueing system with reneging and retention of impatient customers’, International Journal of Operational Research , Vol.17,No.2,pp.333-344.DOI:https://doi.org/10.1504/IJOR.2013.054439

[6] Kumar R and Sharma S K(2013), ‘Economic analysis of M/M/c/N queue with retention of impatient customers’, International Journal of Operational Research ,Vol.5,No.6, pp.709-720.DOI:https://doi.org/10.1504/IJMOR.2013.057488

[7] Kalidass K and GnanarajJ ,Gopinath S(2014), ‘Transient analysis of an M/M/1 queue with a repairable server and multiple vacations’, International Journal of Mathematics in OperationalResearch, Vol.6, No.2. DOI:https://doi.org/10.1504/IJMOR.2014.059522

[8] Lavanya B, Vennila R and Martin sankoh(2022), Mathematical Modelling of M^x /G(a,b)/1 bulk service queue model with two vacations and setup time in ceramic technology, Articleid:2771494,Hindawi.DOI:https://doi.org/10.1155/2022/2771494

[9] Madan K.C and Abu-Dayyeh W(2003), ‘Steady State Analysis of Two $M^x/M(a,b)/1$ QueueModels with Random breakdowns’, International Journal of Information and Management Sciences, Vol. 14, pp.37 – 51. DOI: Not available

[10] MedhiJ(2002), ‘A single server Poisson input queue with a second optional channelQueueing systems’, Vol. 42,pp.239 - 242. DOI: https://doi.org/10.1023/A:1020519830116

[11] MedhiJ(1992),‘Stochastic Models in queueing Theory’ (2nd Ed.), USA: Academic Press.

[12] Rameshkumar E and Jeyakumar S(2016), ‘Analysis of $M^x/G(a, b)/1$ Queue with closedowntime with controllable arrivals during multiple adaptive vacation’, International Journal of Pureand Applied Mathematics, Vol.106, No.5,pp.79-87. DOI: https://doi.org/10.12732/IJPAM.V106I5.8

[13] Rameshkumar E and Poongkodi S(2024), ‘Analysis of General Bulk Queueing Model WithBinomial Service, Set Up Times and Multiple Adaptive Vacation On Empty Queues’,International Journal of Natural Sciences,Vol 15, issue 86, pp.80165-80174. DOI: Not Available

[14] Rakesh kumar and Sapanasharma(2019), ‘Transient analysis of an M/M/c queueing system with retention of reneging customers’, International Journal of Operational Research, Vol. 36, No. 1.pp.78-91. DOI:https://doi.org/10.1504/IJOR.2019.102071

[15] Rahim K H and ThiagarajanM(2023), ‘M/M(a,b)/1 Model of Independent Queueing withControllable Arrival Rates’, Indian Journal of Science and Technology, Vol 16, No.37,pp.3100-310.DOI: https://doi.org/ 10.17485/IJST/v16i37.1259

[16] Senthilnathan B and JeyakumarS(2013), ‘Modelling and analysis of a M^x /G(a,b)/1 queue with multiple vacations, setup time, closedown time and server breakdown without interruption’, International Journal of Operational Research, pp.114-139 Vol. 19, No. 1.DOI:https://doi.org/10.1504/IJOR.2014.057849

[17] Sudhesh R and AzhagappanA(2019), ‘Analysis of an M/M/c queue with heterogeneous servers, balking and reneging’, International Journal of Operational Research, Vol. 36, No. 3.DOI:https://doi.org/10.1504/IJOR.2019.103120

[18] SaratkumarAcharya ,Sara veronica and Cesar Emilio Villarreal (2013), ‘Maximum likelihood estimates in an M/M/c queue with heterogeneous servers’,International Journal of Mathematics in Operational Research, Vol.5, No.4. DOI: https://doi.org/10.1504/IJMOR.2013.054735