A- Level Bijective Single Valued Neutrosophic Graphs and its Applications in Read Recipients in Whatsapp Messages

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Abstract
Objectives: This study aims to introduce a new concept called α – Level Bijective Single Valued Neutrosophic sets and extend this concept to graphs. It also seeks to establish fundamental set- theoretic operations within this framework.
Methods: The study defines basic operations of Bijective Single Valued Neutrosophic sets which include subset, equality, union, intersection and difference. Additionally, the α- level approach is applied to analyze these sets and De Morgan's laws are formulated within this context.
Findings: The paper successfully developes the theoretical foundations of α – Level Bijective Single Valued Neutrosophic sets and demonstrates several results based on the defined operations. it also verifies that De Morgan's laws hold under the proposed structure.
Novelty: The novelty of this work lies in introducing the concept of α – Level Bijective Single Valued Neutrosophic sets and extending it to graph, providing a new perspective and framework for handling uncertainty and indeterminacy in mathematical structures. Also it has an application in social media.

Keywords: Bijective single valued neutrosophic graph, α – Level Bijective Single Valued Neutrosophic Graphs, Strong α – Level Bijective Single Valued Neutrosophic Graphs.

1. Introduction

Real world systems often exhibit uncertainty, vagueness and indeterminacy, especially in domain such as decision making, engineering and social network analysis. To address such complexities, the concept of fuzzy sets was first introduced by Lofti. A.Zadeh in 1965, which provided a mathematical framework to deal with imprecise information[1]. Later, K. Atanassov extended this idea through intuitionistic fuzzy sets by incorporating a degree of non- membership along with membership [2].

Despite these advancements, handling indeterminacy explicitly remained a challenge. To overcome this limitation, Florentin Smarandache introduced the concept of neutrosophic sets, which consider three independent components: truth –membership, indeterminacy- membership and falsity- membership [3, 4]. This theory was further developed into single valued neutrosophic sets by Wang et al, making it more applicable for practical problems [5].

Graph theory, being a powerful tool for modelling relationships, had also been extended into uncertain environments. The integration of fuzzy and intuitionistic fuzzy concepts into graph theory led to the development of intuitionistic fuzzy graphs,as discussed by Gani and Begum[6]. Subsequently, neutrosophic graph theory has gained significant attention, with foundational contributions by researchers such as Broumi et al. and Sahin who explored various structures and properties of single valued and bipolar neutrosophic graphs[7,8,9].

In recent years, several operations and characteristics of neutrosophic graphs have been studied extensively. For instance, Akram and Saahzadi investigated operations on single valued neutrosophic graphs [10], while Ahmed and Alsharafi examined domination concepts in bipolar fuzzy graph operations [11]. The concept of α- level sets, which plays a crucial role in simplifying neutrosophic structures, has been explores by Awolola[12] and futher extended in applications by Das at al., particularly in strong α-level neutrosophic sets[13].

Motivated by these developments, the study of structural properties and operations on neutrosophic graphs has become an active research area. In this context, recent work by D.Rajalaxmi et al. introduced the concept of bijective single- valued neutrosophic graphs and demonstrated their applicability in fraud detection within social network [14, 15]. Such studies highlight the practical relevence of neutrosophic graph models in handling uncertainty in complex systems. Sreelakshmi, T. P. et al studied about Total domination in uniform single valued neutrosophic graph [16].

The present study introduces the concept of α-level bijective single-valued neutrosophic sets and extends it to graph theory. The work establishes fundamental set-theoretic operations, including subset, equality, union, intersection, and difference, within this framework. Furthermore, an α-level approach is employed to analyze these structures, and De Morgan's laws are formulated and verified. The proposed framework not only strengthens the theoretical foundation of neutrosophic sets and graphs but also provides a novel perspective for modelling uncertainty, with potential applications in areas such as social network analysis.

Methodology

This section includes preliminaries which are needed to understand the concepts in detail.

Definition 2.1 [14]:

Let X be a crisp set. A Bijective Single Valued Neutrosophic Set (BSVNS) A is characterized by truth membership function $T_V(P)$, an indeterminacy membership function $I_V(P)$ and a falsity membership function $F_V(P)$ for every point $p \in V$; $T_V(p), I_V(p), F_V(p) \in [0,1]$ such that $T_V(P_i)$ and $T_V(P_j)$, $I_V(P_i)$ and $I_V(P_j)$, $F_V(P_i)$ and $F_V(P_j)$ are distinct respectively, for any *i* and *j*.

Definition 2.2 [14]:

A Bijective Single Valued Neutrosophic Graph (BSVNG) is a triplet $G = (V, \sigma, \mu)$ in which the truth-membership function of vertices and edges are bijective (i.e,) $\sigma_T : V \to [0,1]$ and $\mu_T : E \subseteq V \times V \to [0,1]$ are bijective. Similarly, indeterminacy-membership function of vertices and edges and falsity-membership function of vertices and edges are bijective respectively i.e, $\sigma_I : V \to [0,1]$ and $\mu_I : E \subseteq V \times V \to [0,1]$ are bijective, and $\sigma_F : V \to [0,1]$ and $\mu_F : E \subseteq V \times V \to [0,1]$ are bijective where

$\mu_T(V_i, V_j) < \min(\sigma_T(v_i), \sigma_T(v_j))$, $\mu_I(V_i, V_j) > \max(\sigma_I(v_i), \sigma_I(v_j))$ and

$\mu_F(V_i, V_j) > \max(\sigma_F(v_i), \sigma_F(v_j))$ for all $v_i \in V$ *i , j=1,2...n.*

The following conditions are trivial in BSVNG as in case of SVNG:

i. $0 \le \sigma_T(v_i) + \sigma_I(v_i) + \sigma_F(v_i) \le 3$ for all $v_i \in V$ *(i=1,2…n)*

ii. $0 \le \mu_T(v_i, v_j) + \mu_I(v_i, v_j) + \mu_F(v_i, v_j) \le 3$ for all $v_i, v_j \in V \times V$ *(i,j=1,2…n).*

Definition 2.3:

Let *A* be any Bijective Single Valued Neutrosophic set in a non-empty set *X*. Then the *α-level sets of A* denoted by *(V, α)* and for any *α ∈ [0,1] defined as*

$$(V, \alpha) = \{ x \in X : T_A(x) \ge \alpha,\ I_A(x) + F_A(x) \le 1 - \alpha \}$$

Definition 2.4:

A Bijective Single Valued Neutrosophic Graph (BSVNG) *G = (V, E)* is called *α - Level (or) (G , α) - Level Bijective Single Valued Neutrosophic Graph* if

$$T_V(x) \ge \alpha,\ I_V(x) + F_V(x) \le 1 - \alpha \ \ for\, all\, x \in V$$

$$T_E(xy) \ge \alpha,\ I_E(xy) + F_E(xy) \le 1 - \alpha \ \ for\, all\, xy \in E,\ \alpha \in [0,1]$$

Definition 2.5:

A Bijective Single Valued Neutrosophic Graph *G = (V , E)* is said to be an Empty Bijective Single Valued Neutrosophic Graph if *V = Φ* and *E = Φ*.

Definition 2.6:

If *A* and *B* are two Bijective Single Valued Neutrosophic sets in a non-empty set *X*. Then

1. $A \subseteq B \Leftrightarrow T_A(x) \le T_B(x),\ I_A(x) \ge I_B(x),\ F_A(x) \ge F_B(x)$ *for all* $x \in X$
2. $A = B \Leftrightarrow T_A(x) = T_B(x),\ I_A(x) = I_B(x),\ F_A(x) = F_B(x)$ *for all* $x \in X$
3. $A \cap B = \{ x, \wedge (T_A(x), T_B(x)), \vee (I_A(x), I_B(x)), \vee (F_A(x), F_B(x)) \mid for\ all\, x \in X \}$
4. $A \cup B = \{ x, \vee (T_A(x), T_B(x)), \wedge (I_A(x), I_B(x)), \wedge (F_A(x), F_B(x)) \mid for\ all\, x \in X \}$
5. $A^C = \{ x,\ F_A(x), 1 - I_A(x), T_A(x) \mid for\ all\, x \in X \}$
6. $A \setminus B = \{ x,\ T_A(x) \wedge F_B(x),\ I_A(x) \vee 1 - I_B(x),\ F_A(x) \vee T_B(x) \mid for\ all\, x \in X \}$

Remark 2.7:

If G is empty bijective single valued neutrosophic graph. Then α=1.

1. Results and Discussions

Theorem 3.1

If A is a subset of B. Then *α*-level set of A is a subset of *α*-level set of B.

Proof:

Let $A \subseteq B$. Assume that $x \in (A, \alpha)$

$\Rightarrow T_A(x) \ge \alpha,\ I_A(x) + F_A(x) \le 1 - \alpha$

$\Rightarrow T_B(x) \ge \alpha,\ I_B(x) + F_B(x) \le 1 - \alpha$

[Since, $A \subseteq B \Rightarrow T_A(x) \le T_B(x),\ I_A(x) \ge I_B(x),\ F_A(x) \ge F_B(x)$]

$\Rightarrow x \in (B, \alpha)$

Hence $A \subseteq B \Rightarrow (A, \alpha) \subseteq (B, \alpha)$.

Theorem 3.2:

If A is a Bijective Single Valued Neutrosophic sets in a non-empty set X.
Then $\alpha \geq \beta \Rightarrow (A,\alpha) \subseteq (A,\beta)$.

Proof:

Let $\alpha \geq \beta$. Assume that $x \in (A,\alpha)$

$\Rightarrow T_A(x) \geq \alpha,\ I_A(x) + F_A(x) \leq 1-\alpha$

$\Rightarrow T_A(x) \geq \alpha \geq \beta,\ I_A(x) + F_A(x) \leq 1-\alpha \leq 1-\beta$

$\Rightarrow T_A(x) \geq \beta,\ I_A(x) + F_A(x) \leq 1-\beta$

$\Rightarrow x \in (A,\beta)$

Hence $\alpha \geq \beta \Rightarrow (A,\alpha) \subseteq (A,\beta)$.

Theorem 3.3:
The α – level set of the union of sets is equal to the union of the α – level sets.

Proof:

$(A \cup B,\alpha)$

$=\{x \in X\text{: } T_{A\cup B}(x) \geq \alpha,\ I_{A\cup B}(x) + F_{A\cup B}(x) \leq 1\text{-}\alpha\}$

$=\{x \in X\text{:} \vee \{T_A(x), T_B(x)\} \geq \alpha,\ \wedge \{I_A(x), I_B(x)\} + \wedge \{F_A(x), F_B(x)\} \leq 1\text{-}\alpha\}$

$=\{x \in X\text{:} \{T_A(x) \vee T_B(x)\} \geq \alpha,\ \{I_A(x) + F_A(x)\} \wedge \{I_B(x) + F_B(x)\} \leq 1\text{-}\alpha\}$

$=\{x \in X\text{ :} \{T_A(x) \geq \alpha,\ I_A(x) + F_A(x) \leq 1-\alpha\} \cup \{T_B(x) \geq \alpha,\ I_B(x) + F_B(x) \leq 1\text{-}\alpha\}\}$

$= \{x \in X\text{: } T_A(x) \geq \alpha,\ I_A(x) + F_A(x) \leq 1-\alpha\} \cup$

$\{x \in X\text{: } T_B(x) \geq \alpha,\ I_B(x) + F_B(x) \leq 1-\alpha\}$

So, $(A \cup B,\alpha) = (A,\alpha) \cup (B,\alpha)$.

Theorem 3.4:
The α – level set of the intersection of sets is equal to the intersection of the α – level sets.

Proof:

$(A \cap B,\alpha)$

$= \{x \in X\text{: } T_{A\cap B}(x) \geq \alpha,\ I_{A\cap B}(x) + F_{A\cap B}(x) \leq 1\text{-}\alpha\}$

$= \{x \in X\text{:} \wedge \{T_A(x), T_B(x)\} \geq \alpha, \vee \{I_A(x), I_B(x)\} + \vee \{F_A(x), F_B(x)\} \leq 1\text{-}\alpha\}$

$= \{x \in X\text{: } T_A(x) \wedge T_B(x) \geq \alpha, \{I_A(x) + F_A(x)\} \vee \{I_B(x) + F_B(x)\} \leq 1\text{-}\alpha\}$

$= \{x \in X\text{: } \{T_A(x) \geq \alpha,\ I_A(x) + F_A(x) \leq 1-\alpha\} \cap \{T_B(x) \geq \alpha, I_B(x) + F_B(x) \leq 1\text{-}\alpha\}\}$

$= \{x \in X\text{:} T_A(x) \geq \alpha,\ I_A(x) + F_A(x) \leq 1-\alpha\} \cap$

$\{x \in X\text{: } T_B(x) \geq \alpha,\ I_B(x) + F_B(x) \leq 1-\alpha\}$

Hence $(A \cap B,\alpha) = (A,\alpha) \cap (B,\alpha)$.

Theorem 3.5:

$A = B \Leftrightarrow (A,\alpha) = (B,\alpha),\ \textit{for all}\ \alpha \in [0,1]$.

Proof:

Assume that A=B

$\Rightarrow T_A(x) = T_B(x),\ I_A(x) = I_B(x),\ F_A(x) = F_B(x),\ \textit{for all}\ x \in X$.

$(A,\alpha) = \{x \in X\text{: } T_A(x) \geq \alpha,\ I_A(x) + F_A(x) \leq 1-\alpha\}$

$= \{x \in X\text{: } T_B(x) \geq \alpha,\ I_B(x) + F_B(x) \leq 1-\alpha\}$ [Since, A=B]

$= (B,\alpha)$

Hence $(A,\alpha) = (B,\alpha),\ \textit{for all}\ \alpha \in [0,1]$.

Converse of the theorem, Let us consider for all $\alpha \in [0,1]$, $(A,\alpha) = (B,\alpha)$ but A ≠ B. Suppose x ∈ A such that x ∉ B. There exists an $\alpha \in [0,1]$ for each $x \in (A,\alpha)$. But, x ∉ B implies $x \notin (B,\alpha)$ for each $\alpha \in [0,1]$. Therefore $(A,\alpha) \neq (B,\alpha)$ which is a contradiction to $(A,\alpha) = (B,\alpha)$ for all $\alpha \in [0,1]$.

Hence $A = B \Leftrightarrow (A,\alpha) = (B,\alpha),\ \textit{for all}\ \alpha \in [0,1]$.

Theorem 3.6:
For any bijective single valued neutrosophic sets A_i,

$\left(\bigcup_{i\in J} A_i,\alpha\right) = \bigcup_{i\in J} (A_i,\alpha)$.

Proof:

$$\left(\bigcup_{i\in J}(A_i,\alpha)\right)$$
$$= \{x \in X : T_{\bigcup_{i\in J} A_i}(x) \geq \alpha,\ I_{\bigcup_{i\in J} A_i}(x) + F_{\bigcup_{i\in J} A_i}(x) \leq 1-\alpha\}$$
$$= \{x \in X : \vee_{i\in J} T_{A_i}(x) \geq \alpha,\ \wedge_{i\in J} I_{A_i}(x) + \wedge_{i\in J} F_{A_i}(x) \leq 1-\alpha\}$$
$$= \{x \in X : \vee_{i\in J} T_{A_i}(x) \geq \alpha,\ \wedge\{\Sigma_{i\in J} I_{A_i}(x) + F_{A_i}(x)\} \leq 1-\alpha\}$$
$$= \bigcup_{i\in J}\{x \in X : T_{A_i}(x) \geq \alpha,\ I_{A_i}(x) + F_{A_i}(x) \leq 1-\alpha\}$$
$$= \bigcup_{i\in J}(A_i,\alpha)$$

Hence $\left(\bigcup_{i\in J}(A_i,\alpha)\right) = \bigcup_{i\in J}(A_i,\alpha)$.

Theorem 3.7:
For any bijective single valued neutrosophic sets A_i,
$\left(\bigcap_{i\in J} A_i,\alpha\right) = \bigcap_{i\in J}(A_i,\alpha)$.

Proof:

$$\left(\bigcap_{i\in J}(A_i,\alpha)\right)$$
$$= \{x \in X : T_{\bigcap_{i\in J} A_i}(x) \geq \alpha, I_{\bigcap_{i\in J} A_i}(x) + F_{\bigcap_{i\in J} A_i}(x) \leq 1-\alpha\}$$
$$= \{x \in X : \wedge_{i\in J} T_{A_i}(x) \geq \alpha,\ \vee_{i\in J} I_{A_i}(x) + \vee_{i\in J} F_{A_i}(x) \leq 1-\alpha\}$$
$$= \{x \in X : \wedge_{i\in J} T_{A_i}(x) \geq \alpha,\ \vee\{\Sigma_{i\in J} I_{A_i}(x) + F_{A_i}(x)\} \leq 1-\alpha\}$$
$$= \bigcap_{i\in J}\{x \in X : T_{A_i}(x) \geq \alpha,\ I_{A_i}(x) + F_{A_i}(x) \leq 1-\alpha\}$$
$$= \bigcap_{i\in J}(A_i,\alpha)$$

Hence $\left(\bigcap_{i\in J}(A_i,\alpha)\right) = \bigcap_{i\in J}(A_i,\alpha)$.

Theorem 3.8:
The strong α – level set is always a subset of α – level set.
Proof:

Let $x \in S(A,\alpha)$
$\Rightarrow T_A(x) > \alpha, I_A(x) + F_A(x) < 1-\alpha$
$\Rightarrow T_A(x) \geq \alpha,\ I_A(x) + F_A(x) \leq 1-\alpha$
$\Rightarrow x \in (A,\alpha)$
Hence $S(A,\alpha) \subseteq (A,\alpha)$, for all $\alpha \in [0,1]$.

Theorem 3.9:

If A is a Bijective Single Valued Neutrosophic set in a fixed set X. Let $\beta \in [0,1]$ and $\alpha_i \in [0,\beta)$ for each $i \in J$ where Λ is an indexed set. Then $(A,\beta) \subseteq \bigcap_{i\in J}(A,\alpha_i)$.

Proof:

Let $x \in (A,\beta)$
$\Rightarrow T_A(x) \geq \beta,\ I_A(x) + F_A(x) \leq 1-\beta,\ \textit{for all}\ x \in X$

$\Rightarrow T_A(x) \geq \beta > \alpha_i, I_A(x) + F_A(x) \leq 1-\beta < 1-\alpha_i$ *for all* $x \in X, \alpha_i \in [0,\beta)$

$\Rightarrow T_A(x) \geq \alpha_i, I_A(x) + F_A(x) \leq 1-\alpha_i$ *forall* $x \in X, \alpha_i \in [0,\beta)$

$\Rightarrow x \in (A,\alpha_i),\ \alpha_i \in [0,\beta)$

$\Rightarrow x \in \bigcap_{i \in J} (A,\alpha_i)$

Hence $(A,\beta) \subseteq \bigcap_{i \in J} (A,\alpha_i)$.

Theorem 3.10:

If A is a Bijective Single Valued Neutrosophic set in a fixed set X. Let $\beta \in [0,1]$ and $\alpha_i \in [\beta,1]$ for each $i \in J$ where Λ is an indexed set. Then $\bigcup_{i \in J} (A,\alpha_i) \subseteq (A,\beta)$.

Proof:

Let $x \in \bigcup_{i \in J} (A,\alpha_i)$

Then $x \in (A,\alpha_i)$ for each $i \in J$

$\Rightarrow T_A(x) \geq \alpha_i,\ I_A(x) + F_A(x) \leq 1-\alpha_i$

$\Rightarrow T_A(x) \geq \alpha_i > \beta,\ I_A(x) + F_A(x) \leq 1-\alpha_i < 1-\beta$

$\Rightarrow T_A(x) \geq \beta,\ I_A(x) + F_A(x) \leq 1-\beta$

$\Rightarrow x \in (A,\beta)$

Hence $\bigcup_{i \in J} (A,\alpha_i) \subseteq (A,\beta)$.

De Morgan's laws in Bijective Single Valued Neutrosophic sets:

Theorem 3.11:

$(A \cup B,\alpha)^c = (A,\alpha)^c \cap (B,\alpha)^c$ *for all* $\alpha \in [0,1]$.

Proof:

$(A \cup B,\alpha)^c$

$= \{x \in X\text{: } T_{(A \cup B)^c}(x) \geq \alpha,\ I_{(A \cup B)^c}(x) + F_{(A \cup B)^c}(x) \leq 1\text{-}\alpha\}$

$= \{x \in X\text{: } T_{A^c \cap B^c}(x) \geq \alpha,\ I_{A^c \cap B^c}(x) + F_{A^c \cap B^c}(x) \leq 1\text{-}\alpha\}$

$= \{x \in X\text{: } \wedge\{T_{A^c}(x), T_{B^c}(x)\} \geq \alpha,\ \vee\{I_{A^c}(x), I_{B^c}(x)\} + \vee\{F_{A^c}(x), F_{B^c}\}(x) \leq 1-\alpha\}$

$= \{x \in X\text{: } \wedge\{F_A(x), F_B(x)\} \geq \alpha,\ \vee\{1-I_A(x), 1-I_B(x)\} + \vee\{T_A(x), T_B(x)\} \leq 1-\alpha\}$

$= \{x \in X\text{: } \{F_A(x) \wedge F_B(x)\} \geq \alpha,\ \{1-I_A(x)+T_A(x)\} \vee \{1-I_B(x)+T_B(x)\} \leq 1-\alpha\}$

$= \{x \in X\text{: } F_A(x) \geq \alpha,\ 1-I_A(x)+T_A(x) \leq 1-\alpha\} \cap$
$\{x \in X\text{: } F_B(x) \geq \alpha,\ 1-I_B(x)+T_B(x) \leq 1-\alpha\}$

$= (A,\alpha)^c \cap (B,\alpha)^c$

Hence $(A \cup B,\alpha)^c = (A,\alpha)^c \cap (B,\alpha)^c$.

Theorem 3.12:

$(A \cap B,\alpha)^c = (A,\alpha)^c \cup (B,\alpha)^c$.

Proof:

$(A \cap B,\alpha)^c$

$= \{x \in X\text{: } T_{(A \cap B)^c}(x) \geq \alpha,\ I_{(A \cap B)^c}(x) + F_{(A \cap B)^c}(x) \leq 1\text{-}\alpha\}$

$= \{x \in X\text{: } T_{A^c \cup B^c}(x) \geq \alpha,\ I_{A^c \cup B^c}(x) + F_{A^c \cup B^c}(x) \leq 1\text{-}\alpha\}$

$= \{x \in X\text{: } \vee\{T_{A^c}(x), T_{B^c}(x)\} \geq \alpha,\ \wedge\{I_{A^c}(x), I_{B^c}(x)\} + \wedge\{F_{A^c}(x), F_{B^c}(x)\} \leq 1-\alpha\}$

$=\{x \in X: \vee\{F_A(x), F_B(x)\} \geq \alpha, \ \wedge\{1-I_A(x), 1-I_B(x)\} + \wedge\{T_A(x), T_B(x)\} \leq 1-\alpha\}$

$=\{x \in X: \{F_A(x) \vee F_B(x)\} \geq \alpha, \ \{1-I_A(x)+T_A(x)\} \wedge \{1-I_B(x)+T_B(x)\} \leq 1-\alpha\}$

$= \{x \in X: F_A(x) \geq \alpha, \ 1-I_A(x)+T_A(x) \leq 1-\alpha\} \cup \{x \in X: F_B(x) \geq \alpha, \ 1-I_B(x)+T_B(x) \leq 1-\alpha\}$

$=(A,\alpha)^C \cup (B,\alpha)^C$

Hence $(A \cap B,\alpha)^C = (A,\alpha)^C \cup (B,\alpha)^C$.

Remark 3.13:

If $\{V_i \mid i \in J\}$ be a family of Bijective Single Valued Neutrosophic sets and $\alpha \in [0,1]$. Then the De Morgan's Laws hold:

i. $(\bigcup_{i \in J} V_i, \alpha)^C = \bigcap_{i \in J} (V_i, \alpha)^C$.

ii. $(\bigcap_{i \in J} V_i, \alpha)^C = \bigcup_{i \in J} (V_i, \alpha)^C$.

Theorem 3.14:

If A $\subset$B. Then complement of the α - level set of B is contained in complement of the α - level set of A.

Proof:

Given that $A \subseteq B$

Let $x \in (B,\alpha)^C$

$\Rightarrow F_B(x) \geq \alpha, 1-I_B(x) + T_B(x) \leq 1-\alpha$

Since $A \subseteq B \Rightarrow T_A(x) \leq T_B(x), \ I_A(x) \geq I_B(x), \ F_A(x) \geq F_B(x)$

$\Rightarrow F_A(x) \geq \alpha, 1-I_A(x) + T_A(x) \leq 1-\alpha$

$\Rightarrow x \in (A,\alpha)^C$

Hence $A \subseteq B \Rightarrow (B,\alpha)^C \subseteq (A,\alpha)^C$.

Theorem 3.15:

The α- level set of the difference of two sets A and B is equal to the α- level set of the intersection of A with the complement of B.

Proof:

$(A \setminus B, \alpha) = \{x \in X: T_A(x) \wedge F_B(x) \geq \alpha, \ I_A(x) \vee 1-I_B(x) + F_A(x) \vee T_B(x) \leq 1-\alpha\}$

$= \{x \in X: \wedge\{T_A(x), F_B(x)\} \geq \alpha, \ \vee\{I_A(x), 1-I_B(x)\} + \{F_A(x), T_B(x)\} \leq 1-\alpha\}$

$= \{x \in X: T_A(x) \geq \alpha, \ I_A(x) + F_A(x) \leq 1-\alpha\} \cap \{F_B(x) \geq \alpha, \ 1-I_B(x)+T_B(x) \leq 1-\alpha\}$

$= (A \cap B^C, \alpha)$

Hence $(A \setminus B, \alpha) = (A \cap B^C, \alpha)$.

1.2 Applications of α – level bijective single valued neutrosophic graphs in read recipients in WhatsApp messages:

End-to-end encryption is used in WhatsApp to ensure communication between users. It guarantees that the message remain private and can only be accessed by the sender and intended recipient. No third party, including WhatsApp itself, can read, listen to or share these messages. This level of security is achieved because the messages are encrypted in such a way that only the communicating users can decrypt them.

In this context, the process of identifying read recipients in WhatsApp can be modelled using BSVNG.

Consider a BSVNG, the vertices are represented as WhatsApp users (Senders and Receivers). A vertex membership values are represented as

T- read messages

I - delivered messages

F - sent messages.

The edges are represented as message check marks. The edge membership values are represented as follows:

T – Two blue check mark ✓✓(i.e.) Message read.

I – Two grey check mark ✓✓(i.e.) Message delivered, not read yet.

F – One grey check mark ✓(i.e.) Message sent, not delivered yet.

Consider a group chats in WhatsApp:

Here the vertex set is $V = (v_1, v_2, v_3, v_4)$ represents a sender(v_1) and receivers (v_2, v_3, v_4). $E = (e_1, e_2, e_3, e_4)$ represents a message check marks between the sender and receivers to check the read recipients in Group chat.

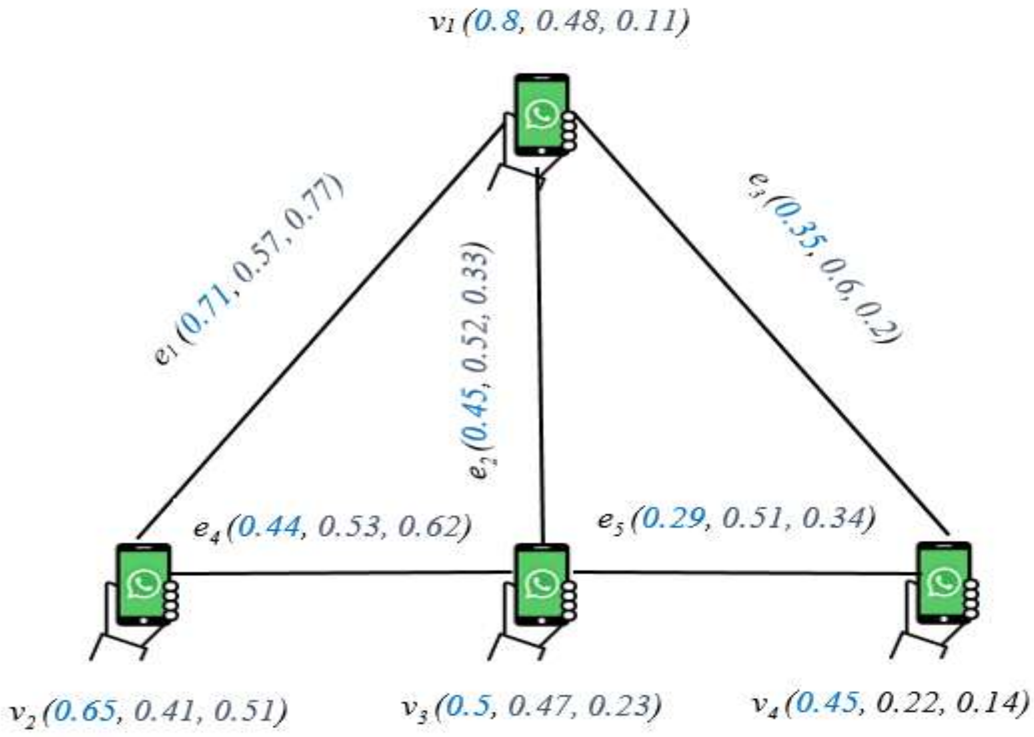

Figure 1 Group chat in WhatsApp

The read status of recipients is analysed within one hour time interval.
Let $\alpha = 0.1$, (i.e.) 10 percent of one hour is 6 minutes.

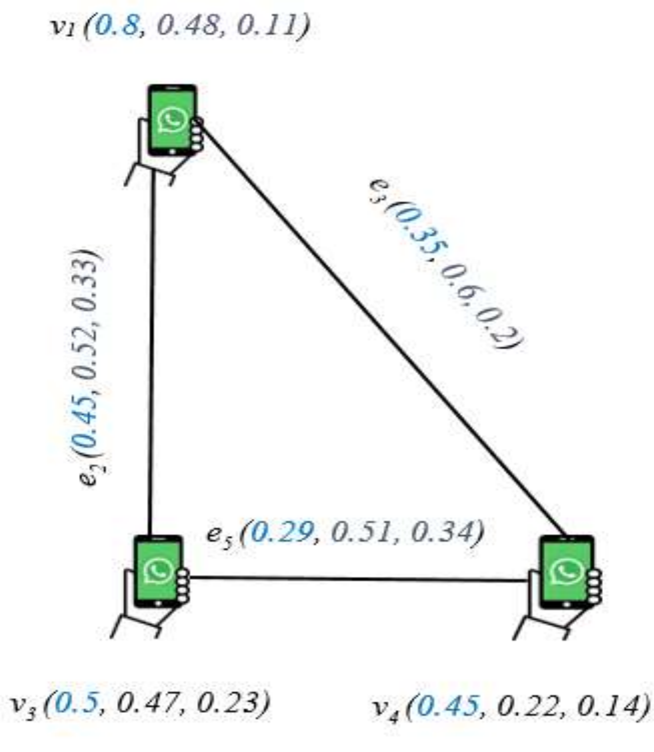

Figure 2 Group chats

Let check who saw the message in 6 minutes.

In this case, a group chat in WhatsApp v_2 not delivered the message but v_3 and v_4 read the message in 6 minutes. (i.e.) They chat with each other.
Now, to check the read recipients during one hour.
In this case, let α = 0.3, (i.e.) 30 percent of one hour is 18 minutes.

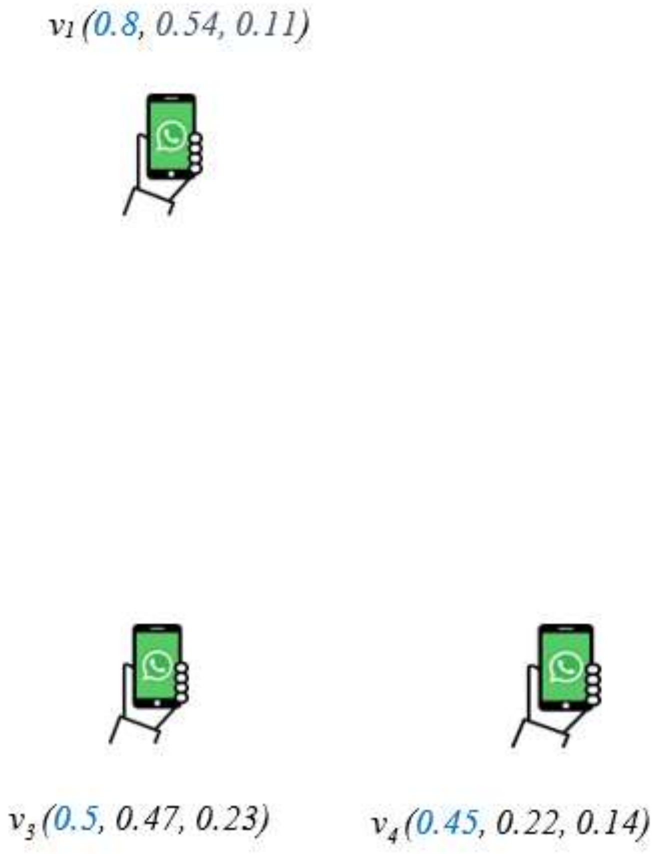

Figure 3 group chats

Let check who saw the message in 18 minutes.
In this case, a group chat in WhatsApp v_2 not delivered the message butv_3 *and*v_4 remain their group but no interactions between sender and receivers in 18 minutes. (i.e.) The group members are offline.
Let α = 1, no senders and receivers in their group.
(i.e.) The sender and receivers are offline.
Similarly, we can find the different level of read recipients using BSVNG.

Conclusion

In this study, α – level bijective single valued neutrosophic sets and graphs has been defined and several results have been established using basic operations. The Relationship between α – level and strong α – level bijective single valued neutrosophic sets has been analyzed. Also, some complement properties are defined and examined. Furthermore, the applicability of the proposed concept has been illustrated through a practical example in WhattsApp communication. In future, some more properties of α – level bijective single valued neutrosophic graphs can be explored.

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